Digital Signal Extraction by Means of Nonlinear Stochastic Filtration

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The results of noise immunity analysis for digital communication systems using methods of nonlinear filtering are given. Nonlinear filtration is based on stochastic resonance effect. The stochastic resonance is given to a phenomenon that is manifest in nonlinear systems where by generally feeble input information (such as a weak signal) can be amplified and optimized by the assistance of noise. The stochastic resonance has been observed in a large variety of systems, including bistable ring lasers, semiconductor devices, chemical reactions, and mechanoreceptor cells in the tail fan of a crayfish. Numeral simulation of response at affecting input of the system on additive mixture of harmonic signal and white Gaussian noise are given. Amplitude spectrum of this output signal has been investigated. Results of the output signal-to-noise ratio calculation of the stochastic filter for the additive sum of a harmonic signal and white Gaussian noise for different values of the input noise dispersion are given. It is shown that the output signal-to-noise ratio of the system will peak at a certain value of noise intensity under a action of the input signal and noise. It is shown that the stochastic resonance effect provides separation of a digital signal from the white Gaussian noise. The comparative analysis of noise immunity of the matched filter and nonlinear stochastic filter for input square pulses are given. The effects of signal distortions in nonlinear processing with a stochastic filter are considered. Calculations of the coefficient of nonlinear distortions of a rectangular pulse are performed. It is shown that nonlinear distortions lead to a decrease in the signal-to-noise ratio at the output of the filter.

Key words: stochastic resonance; signal-to-noise ratio; filter; nonlinear stochastic filter; matched filter; digital signal; dispersion; white Gaussian noise; nonlinear distortions

Introduction

At present time digital communication systems are widely used throughout in the world. One advantage of digital information is that it tends to be far more resistant to transmitted and interpreted errors than information symbolized in an analog medium.

The digital systems are basis of modern telecommunication. Basic advantage of the digital systems is high noise immunity, and also possibility to pass and process the large volumes of information. At the same time, a height over of number of the simultaneously functioning information systems inevitably bring to intensifying of problem of compatibility and noise immunity that can not be decided exceptionally by the methods of noise immunity coding. The methods of linear filtration aren't sufficient, especially in broadband communication networks [1–3].

The search of new methods of signal processing on a background intensive Gaussian noise urged on development of vehicle of stochastic filtration, that is based an stochastic resonance effect.

1 Concept of Stochastic Resonance

Stochastic resonance (SR) takes place in non-linear systems driven by an information signal and noise [4]. Output signal-to-noise ratio is significantly improved at a certain optimal noise level in the regime of SR.

In the beginning we will consider effect of standing out of the harmonic signal. The equation describing SR effect is given by [4, 5]

$$dy/dt = y(t) - y^3(t) + x(t), \tag{1}$$

where $x(t) = s(t) + n(t)$ – is an input process being additive mixture of harmonic signal and normal noise; $y(t)$ – output signal.

The numeral simulation of response at affecting input of the system on additive mixture of harmonic signal $s(t)$ and white Gaussian noise $n(t)$, illustrating model (1), resulted on Fig. 1.

The figure 1 shows that output noise power decreased, but the output signal waveform differs from harmonic.

Input signal (black), a harmonious signal of frequency $f = 0.05$ Hz (green), dispersion of input noise $D = 0.01$
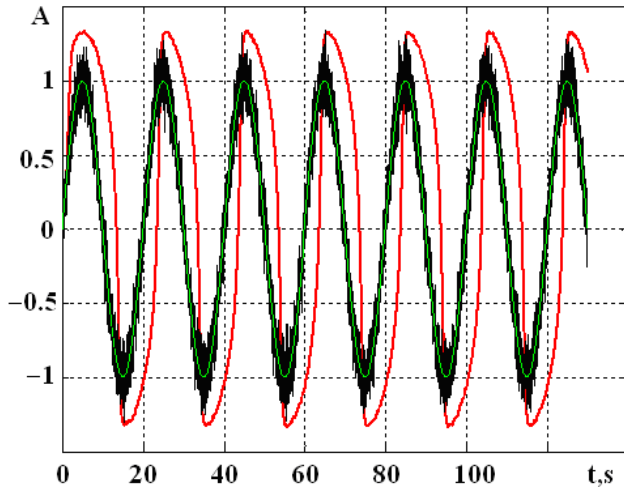
Fig. 1. Standing out of signal from additive signal–noise mixture(red line).

It is explained by nonlinearity of system which is characterized by effect of SR [6,7].

Unlike linear systems, in which the energy spectrum at the output follows input energy spectrum, output spectrum of the non-linear system has a more complicated structure [6,7].

Signal and noise are independent in a linear system. The output of the non-linear device forms new spectra components due to the interaction of the components of the input process. Moreover, the type of non-linear transformation and the statistical characteristics of the input signal determine the type and intensity of the additiona component.

Output power spectrum of the non-linear device if the input is additive mix of the sine signal and noise can be writen as [6,7]:

$F_{SxS}(\omega)$ — is the beats between the components of the signal and its harmonics (a discrete part of the spectrum);

$F_{NxN}(\omega)$ — is the beats of noise components (continuous component of the spectrum);

$F_{SxN}(\omega)$ — is the mutual beats of signal components and noise (continuous component of the spectrum).

The discrete part of the spectrum may contain the spectra line at zero frequency (DC component at the output), which is also determined by the beats of the signal components and noise. Consequently, the energy spectrum of the output of the nonlinear device is determined as [6–9]:

$$F(\omega) = F_{SxS}(\omega) + F_{SxN}(\omega) + F_{NxN}(\omega).$$

We will consider an amplitude spectrum of the output signal given on Fig. 1.

Odd harmonics were in the output spectrum as a result of non-linear conversion (1) which make a powerful contribution to the signal-to-noise ratio (Fig. 2).
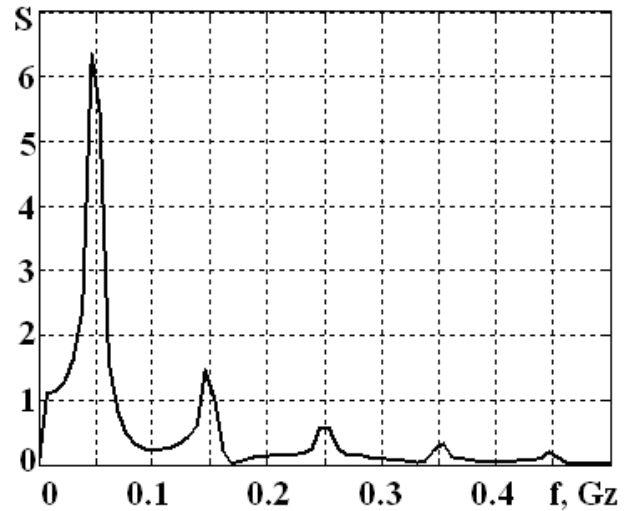

Fig. 2. Amplitude spectrum of the output signal

Practically the most convenient power indicator of the output signal is the signal–to–noise ratio (SNR).

Having solve equation (1) numerically, let's define the output SNR as a function of dispersion D (Fig. 3). The dependence has non-linear character. The local maximum is watched in case of
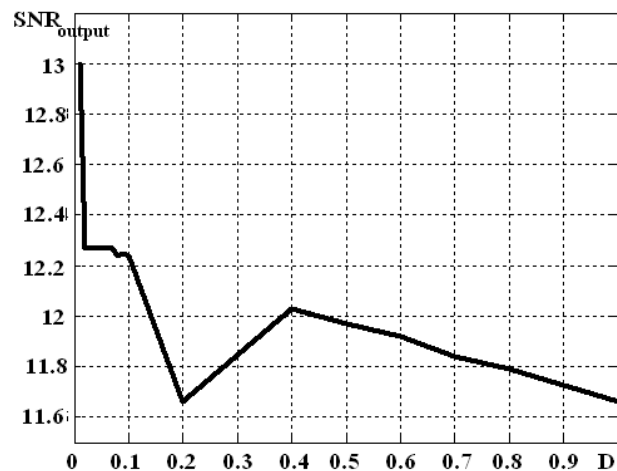
$$D = 0.4$$

.

Fig. 3. SNR dependence on the input noise dispersion (frequency of the input harmonic signal $- f = 0.025$ Hz)

In [10] output SNR dependences are given from the input harmonic signal frequency. This dependence also has non-linear character. the SNR sharply decreases with frequencies over 1 Hz, thus the stochastic resonator is the low pass filter. The SR is characterized by concentrating most of the noise energy into the low-frequency region. That is to say, white noise energy that distributes uniformly in the whole spectrum will mostly be accumulated into low frequencies by the nonlinear bistable system. The energy concentration then leads to the SR phenomenon for the low-frequency driving component.

Increase of signal separation efficiency at high frequency it is possible to achieve frequency displacement to other area [11]. In this article, by means of complex transformations, the transition from general SR equation

$$\frac{dx}{dt} = ax - bx^3 + A\sin(2\pi f_0 t + \varphi) + \sqrt{2D}\xi(t) \quad (2)$$

to equation of another mode has been done

$$\frac{dy}{dt} = y - y^3 + \sqrt{\frac{b}{a^3}}$$
$$\times \left[A\sin\left(\frac{2\pi f_0 \tau}{a} + \varphi\right) + \sqrt{2D}\xi\left(\frac{\tau}{a}\right) \right]. \quad (3)$$

Equation (3) is the normalized form of Eq. (2). They can be considered to be the same one in nature. From these two equations, it can be seen that the frequency of the driving signal is normalized to be $1/a$ times of the original frequency in the new model. The frequencies of the noise are also normalized in the same form. Therefore, the model of Eq. (3) can be used for detection of weak signal with a high frequency. With the normalized scale transformation, choosing a large parameter a can normalize a high frequency (> 1 Hz) to be much smaller than one, which hence satisfies the requirement of the classical SR.

It is obvious that the same result may be obtained by means of normalization of temporal counts of signal.

The analysis of temporal dependences and signals spectra showed that stochastic filtering (SF) there are non-linear distortions caused by non-linear process of conversion of an input signal. The coefficient of harmonic distortion is defined for the quantitative assessment of non-linear distortions [12]:

$$k = \frac{\sqrt{I_2^2 + I_3^2 + I_4^2 + ...}}{I_1},$$

where $I_k - k$-th harmonicas amplitude.

Subjective appraisals of sound signals show that non-linear distortions are practically not felt untill the coefficient of harmonicas does not exceed 1% [12].

It is obvious that the coefficient of harmonicas distortion has to be connected with the level of a noise at stochastic filtration. Coefficient of harmonicas distortion is calculated (Fig. 4).
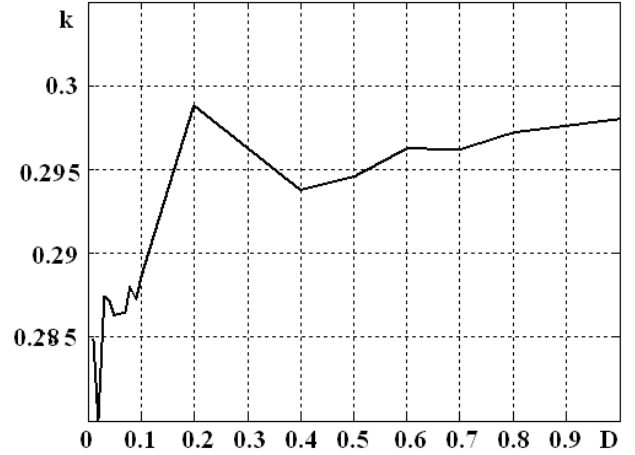

Fig. 4. Dependence of harmonicas distortion coefficient on the noise dispersion (frequency of the input harmonic signal $- f = 0.025$ Hz)

The coefficient of harmonicas distortion is of quite great in this case. The analysis of fig. 3, 4 shows that the coefficient of harmonicas distortion and the SNR at the output are connected. At the $D = 0.2$ the SNR minimum and maximum of coefficient of harmonicas distortion is observed, and at $D = 0.4$ − the SNR maximum and minimum of coefficient of harmonicas distortion take place.

The enrichment of the signal spectrum by harmonics acts equivalent to an increase in the noise level, which leads to a decrease of the signal-to-noise ratio.

2 Extraction of a rectangular impulse

It is known that the model of a rectangular impulse is basic in the theory of information and coding [1–3].

We will consider a rectangular impulse $s(t)$ as a useful input signal. The numerical solution of the equation (1) illustrates effective standing out of pulse signals from mixture with noise of high intensity (Fig. 5). Essential increase of the output SNR of stochastic nonlinear filter can be noted even visually. The signal extraction take place even for very small signal-to-noise ratios on the input. In addition, one can note the effect of smoothing the fronts of a rectangular pulse

Filtration in the transmitter and the channel usually leads to distortion of the impulses sequence caused by an intersymbol interference, therefore, these symbols can't be allocated and detected [13]. The accepting filter has to restore an impulse with the greatest possible SNR and without intersymbol interference. Such filter is the matched filter (MF) which is widely used in digital communication [9].

The matched filter is the linear device designed to give greatest possible output SNR.

The input signal of the digital MF consists of a useful signal $s(t)$ and noise $n(t)$; signal spectral

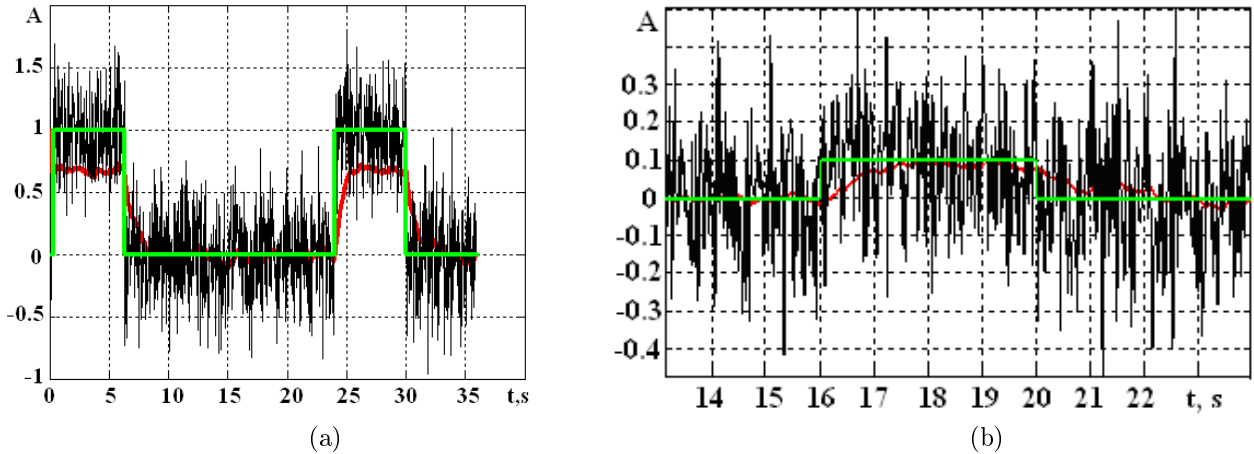(a)   (b)

Fig. 5. Dependence of a useful digital signal (green line), additive mixure of a signal and noise (black line), a signal at the exit (red line). Length of impulses — 6 s. (a) – the input SNR = 10 (on power); (b) – the input SNR = 0.4 (on power).

width $W = 1/2T$, where T – duration of transmission of the symbol. Thus, the minimum Nyquist frequency is equal $f_S = 2W = 1/T$, and time selection T_S will be no more than a transmission time of the symbol. Sampling is made with a frequency of 4 times exceeding the minimum Nyquist frequency in real systems [1].

The output SNR of the MF we will determine by a formula [9]

$$SNR = \frac{2E_S(t_0)}{W_0}$$

, where $E_S(t_0)$ – energy of a signal in time point t_0 ; W_0 – double sided power spectral density of noise.

The comparative analysis of the output SNR of MF and CR shown in Fig. 6 at various lengths of an impulse. It is evidently visible, that CR gives higher output SNR in comparison from MF.
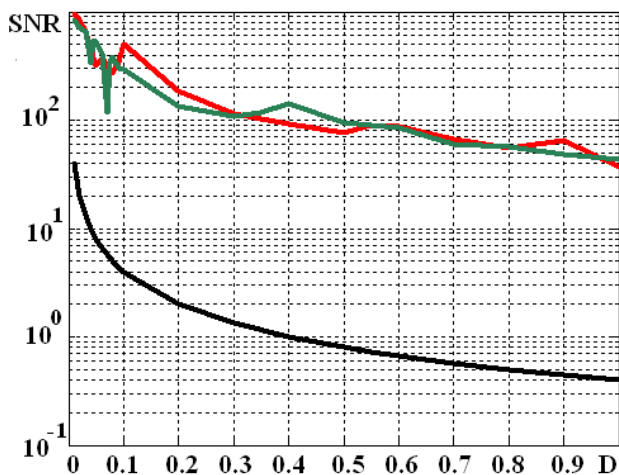

Fig. 6. SNR dependence the SF (red line-duration retangular impulse $T = 5s$; green- $T = 6s$) and the MF (black line $- T = 5s$) on an input noise dispersion

Fig. 5 also shows that output SNR dependence of the SF from an input noise dispersion is nonlinear

for rectangular impulses and has a local maximum in a vicinity of a point $D = 0.1$.

Conclusions

In this paper, based on the results of modeling the effect of SR, it is shown that it is possible to efficiently extract a digital signal from an additive mixture with Gaussian noise of high intensity.

The essential ingredients for SR consist of a nonlinear system, a weak signal, and a source of noise. Using the nonlinear system, the output SNR of the system will peak at a certain value of noise intensity under an action of the input signal and noise. When SR occurs, a certain fraction of the noise energy is transferred to a weak signal and greatly strengthens its intensity. In other words, noise can play a constructive and useful role in nonlinear systems.

Use of a nonlinear stochastic filtration allows reducing significantly output noise, but the form of a output signal significantly differs from harmonious. It speaks of nonlinearity of this system.

It is shown that stochastic filtration is accompanied by nonlinear distortions of the input signal. The output signal shows an increase in odd harmonics.

Use of effect of SR to rectangular pulses showed advantage in comparison with the MF which is widely applied in digital communication. Output SNR dependence of the SF on an input noise dispersion is nonlinear for rectangular impulses and has a local maximum in a vicinity of a point $D = 0.1$. A comparative analysis of the signal-to-noise ratio at the output of a linear matched filter and a stochastic filter is carried out. It is shown that the stochastic resonance effect provides more efficient separation of the digital signal from the additive mixture with Gaussian noise.

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Виділення цифрового сигналу за допомогою нелінійної стохастичної фільтрації

Харченко О. І., Карташов В. М.

Наведено результати аналізу завадостійкості цифрових систем зв'язку з застосуванням методів нелінійної фільтрації. Показано, що ефект стохастичного резонансу забезпечує виділення цифрового сигналу на тлі білого гаусового шуму. Наведено результати розрахунку відношення сигнал/шум на виході стохастичного фільтра для моделі сигналу у вигляді адитивної суміші гармонійного сигналу і білого гаусового шуму при різних значеннях дисперсії вхідного шуму. Зроблено порівняльний аналіз завадостійкості узгодженого фільтра і стохастичного фільтра в разі прямокутних імпульсів на вході. Розглянуто результати розрахунку нелінійних спотворень сигналу на виході стохастичного фільтра.

Ключові слова: стохастичний резонанс; співвідношення сигнал-шум; фільтр; нелінійний стохастичний фільтр; узгоджений фільтр; цифровий сигнал; дисперсія; білий гаусовий шум; нелінійні спотворення

Выделение цифрового сигнала с помощью нелинейной стохастической фильтрации

Харченко О. И., Карташов В. М.

Приведены результаты анализа помехоустойчивости цифровых систем связи с применением методов нелинейной фильтрации. Показано, что эффект стохастического резонанса обеспечивает выделение цифрового сигнала на фоне белого гауссовского шума. Приведены результаты расчета отношения сигнал/шум на выходе стохастического фильтра для модели сигнала в виде аддитивной смеси гармонического сигнала и белого гауссовского шума при различных значениях дисперсии входного шума. Произведен сравнительный анализ помехоустойчивости согласованного фильтра и стохастического фильтра в случае прямоугольных импульсов на входе. Рассмотрены эффекты искажений сигнала при нелинейной обработке стохастическим фильтром. Выполнены расчеты коэффициента нелинейных искажений прямоугольного импульса. Показано, что нелинейные искажения приводят к снижению уровня отношения сигнал/шум на выходе фильтра.

Ключевые слова: стохастический резонанс; отношение сигнал/шум; фильтр; нелинейный стохастический фильтр; согласованный фильтр; цифровой сигнал; дисперсия; белый гауссовский шум; нелинейные искажения