Visnyk NTUU KPI Seriia – Radiotekhnika Radioaparatobuduvannia, 2021, Iss. 86, pp. 58–67

УДК 621.373.826.032:534.232.082.73

Mathematical Modelling of Rod-Type Piezo-Electric Transducers for Acoustoelectronic Devices

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The work is devoted to the peculiarities of the construction and study of mathematical models of rod-type piezoelectric transducers, which are widely used in various acoustoelectronic devices (hydroacoustic means of target detection, ultrasonic non-destructive testing, medical diagnostics, etc.). In contrast to the existing mathematical models of piezoelectric transducers (based on amplitude-phase dependences, resonant piezoelectric transducers, equivalent circuits, etc.), the proposed mathematical model makes it possible to establish a dependence, which is a mathematical description of the acoustic coupling that exists in a solid piezoceramic rod between wave fields on its various areas. An algorithm for calculating a mathematical model of rod-type piezoelectric transducers is presented and based on the determination of the transformation ratio in the case of the inverse piezoelectric effect. Analytical dependencies, which make it possible to determine the electrical impedance and the amplitude value of the potential in the electrical circuit of the piezoelectric transducer, are obtained. It is shown that these dependencies underlie the expression for determining the transformation ratio, which is a mathematical model of a rod piezoelectric transducer. At the same time, the principle of operation of such a transducer provides for the use of longitudinal vibrations in a prismatic rod. The results of the mathematical modelling are presented on the example of a rod transducer with a square cross-section made of piezoelectric ceramics of the PZT type (plumbum zirconate titanate). The performed comparisons of the calculated and experimentally obtained values of the frequency dependence of the modulus of the transformation ratio of the piezoceramic transducer showed a high convergence between them (the discrepancy between the results of mathematical modelling and the experimentally obtained data for the same value of the operating frequency does not exceed 8.5%).

Key words: piezoelectric transducer; acoustoelectronics; mathematical model; electrical signal generator

DOI: 10.20535/RADAP.2021.86.58-67

Introduction

Recently, acoustoelectronic devices have found more and more widespread use due to their miniaturization, multifunctionality, and the possibility of integrating the manufacturing technologies of these devices with microelectronic and microcircuitry technologies. At the same time, the increase in the operational manufacturability of such devices and the versatility of manufacturing technologies leads to a decrease in their cost, which increases the demand for them in various fields of science and technology. At the same time, the concept of sustainable development and the "Industry 4.0" initiatives require the use of renewable technologies and materials that bring biological, environmental and economic safety. The most promising materials for acoustoelectronic devices from the point of view of "Industry 4.0", according to the authors of [1], are piezoelectrics.

So, today, synthetic piezoelectrics (piezoceramics) are used in location-type devices (hydroacoustic target detection, ultrasonic non-destructive testing, medical diagnostics), in primary transducers of electrical measurement systems for non-electrical quantities (information sensors in systems for automatic monitoring and control of objects and technological processes) [2]. This should also include polarized ferroelectrics, which are used to create various microelectromechanical structures (MEMS), a new link in instrumentation, which has been developing more than vigorously since the mid-eighties of the twentieth century [3].

However, the variety of practical applications of piezoelectrics should be accompanied by appropriate methodological (scientific) support. These are methods of calculation, design and, especially, modelling of piezoelectric transducers for various purposes [4].

1 The relevance of the study based on the analysis of recent publications

Many publications are devoted to the construction and study of mathematical models of piezoelectric transducers. However, many of them describe only the processes occurring in a piezoelectric element with a fully electroded surface. For example, in [5], studies of forced vibrations of a piezoceramic plate are carried out taking into account the viscoelasticity of the material at the frequency of the main resonance under the action of an external mechanical harmonious load for cases of rigid fixation and hinge support of the plate contour. In [6], studies of the vibration characteristics of a thin piezoelectric ceramic disk with different ratios of its dimensions are presented. The works [7] and [8] discuss the problem of determining the spectrum of natural frequencies and modes of vibration. In [9], expressions for the instantaneous power are derived. In [10], a mathematical model of resonant piezoelectric transducers with fixed and free ends is developed. A mathematical model of the state of a piezoelectric with a gradient excitation field in the plane of the crystal element is presented in [11]. In [12], the main parameters of piezoelectric ceramics are determined by measuring the maximum and minimum conductivity. The work [13] investigates the amplitude-phase dependences for radial displacements, the sum of principal stresses and admittance components in the vicinity of resonant and antiresonant frequencies. The principles of calculation of the piezoelectric elements with partial electrodes covering are investigated in work [14].

The works considered above are not united by any systematic approach, have the character of scattered episodes, on the basis of which it can be argued that at present there is a need to create an integral methodology for constructing mathematical models of piezoelectric transducers, which could be used as a theoretical basis for calculating their characteristics and parameters.

There are also a number of works based on the use of equivalent circuits [15–18]. Mathematical models of piezoelectric transducers built based on the analysis of the so-called equivalent circuits do not take into account the obvious fact that the motion of material particles of a piezoelectric disk must satisfy the second and third laws of Newton. Which is guaranteed that they are inadequate to real objects and those physical processes occurring in them [19].

Thus, the relevance of the development of physically meaningful mathematical models of rod-type piezoelectric transducers for acoustoelectronic devices remains now.

2 Statement of the problem of modelling a rod piezoelectric transducer

Consider the design of the transducer, which is shown in Fig. 1. A prismatic rod of length L with a rectangular cross-section $\alpha \times b$ is made of piezoelectric ceramics of the PZT type (plumbum zirconate titanate) polarized along the Ox_3 axis. The bottom surface x_3=0 of the rod is electroded and grounded. On the top surface $x_3 = a$ there are two electrodes (positions 1 and 2 in Fig. 1), which form the primary and secondary electrical circuits of the piezoelectric transducer (piezotransformer). The primary circuit consists of a source of the electrical potential difference $U_1 e^{i\omega t}$ (U_1 is the amplitude value of the electrical potential difference; $i = \sqrt{-1}$ is the imaginary unit; ω is the cyclic frequency of the change in the sign of the potential difference; t is time) with the output electrical resistance Z_1 and electrode 1. The secondary electrical circuit of the piezotransformer consists of electrode 2 (position 2 in Fig. 1) and an electrical load Z_2, on which the potential difference $U_2 e^{i\omega t}$ is released.

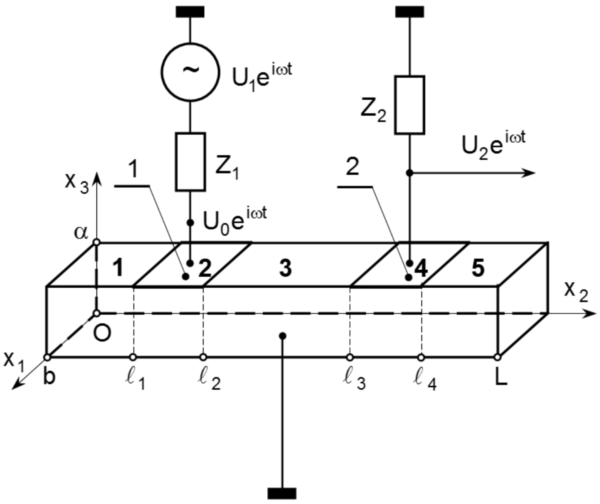

Fig. 1. Schematic presentation of the construction of rod piezoelectric transducer

3 Construction of mathematical model of rod-type piezoelectric transducers

Obviously, the system function (mathematical model) of the piezoelectric transformer is the transformation ratio $K(\omega, \Pi)$ (the symbol Π in the list of arguments of the complex-valued function $K(\omega, \Pi)$ means a set of geometric, physical-mechanical and electrical parameters of the piezoelectric transformer), which is determined in the following way

$$K(\omega, \Pi) = \frac{U_2}{U_1}. \quad (1)$$

When applying to electrode 1 (position 1 in Fig. 1) the difference of electric potentials from an external source the forced longitudinal oscillations of material particles of the piezoelectric arise in the region $l_1 \leq x_2 \leq l_2$ of piezoceramic rod through the inverse piezoelectric effect. Thanks to elastic bonds, all material particles (elementary volumes of a deformed solid) of piezoceramic rod are involved in the process of longitudinal harmonic vibrations. As a result of the direct piezoelectric effect, an electric charge $Q_2(\omega)$ appears on the electrode 2 of the secondary electrical circuit, which with its electric field sets in motion free carriers of electricity in the conductor of the secondary electrical circuit. Electric current $I_2(\omega) = -i\omega Q_2(\omega)e^{i\omega t}$, flowing through the conductor in the secondary electrical circuit, forms on the electrical load Z_2 the difference of electrical potentials $U_2 e^{i\omega t} = -i\omega Z_2 Q_2(\omega)e^{i\omega t}$. Thus

$$U_2 = -i\omega Z_2 Q_2(\omega). \tag{2}$$

The electric charge $Q_2(\omega)$ is determined through the axial component of the electric induction vector $D_3^{(4)}(x_k)$ in a piezoelectric deformed in the region $l_3 \leq x_2 \leq l_4$ as

$$Q_2(\omega) = \int_{\ell_3}^{\ell_4}\int_0^b D_3^{(4)}(x_1, x_2, \alpha)\, dx_1 dx_2. \tag{3}$$

As follows from the equations of the physical state of a deformed piezoelectric [20], the components of the electric induction vector are determined by the components of the elastic strain tensor. The degree of influence of one or another component on the electrical state of a deformed piezoelectric is determined by the components of the matrix of piezoelectric modules $e_{k\beta} \equiv e_{kij}$.

With the above method of electric polarization of a piezoceramic rod, the matrices of its material constants have the following design:

– Matrix of elastic moduli $c^E_{\beta\lambda}$, which are experimentally determined in the mode of constancy (equality to zero) of the electric field strength (symbol E) in the volume of the deformable piezoelectric

$$\left|c^E_{\beta\lambda}\right| = \begin{vmatrix} c^E_{11} & c^E_{12} & c^E_{13} & 0 & 0 & 0 \\ & c^E_{22} & c^E_{23} & 0 & 0 & 0 \\ & & c^E_{33} & 0 & 0 & 0 \\ & & & c^E_{44} & 0 & 0 \\ & & & & c^E_{55} & 0 \\ & & & & & c^E_{66} \end{vmatrix}, \tag{4}$$

where $c^E_{11} = c^E_{22} \neq c^E_{33}$; $c^E_{12} = c^E_{13} = c^E_{23}$; $c^E_{44} = c^E_{55}$; $c^E_{66} = \left(c^E_{11} - c^E_{12}\right)/2$;

– Matrix of piezoelectric moduli $e_{k\beta}$

$$|e_{k\beta}| = \begin{vmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{vmatrix}, \tag{5}$$

where $e_{15} = e_{24} = (e_{33} - e_{31})/2$; $e_{31} = e_{32} \neq e_{33}$;

– Matrix of dielectric permittivities χ^ε_{ij}, whose elements are experimentally determined in the mode of constancy (equality to zero) of elastic deformations (symbol ε) in the volume of the investigated piezoelectric

$$\left|\chi^\varepsilon_{ij}\right| = \begin{vmatrix} \chi^\varepsilon_{11} & 0 & 0 \\ & \chi^\varepsilon_{22} & 0 \\ & & \chi^\varepsilon_{33} \end{vmatrix}, \tag{6}$$

where $\chi^\varepsilon_{11} = \chi^\varepsilon_{22} \neq \chi^\varepsilon_{33}$.

Since shear deformations in the considered piezoceramic rod (Fig. 1) are absent, then the generalized Hooke's law for different sections of the rod can be written as follows

$$\sigma^{(m)}_{11}(x_k) = c^E_{11}\varepsilon^{(m)}_{11}(x_k) + c^E_{12}\varepsilon^{(m)}_{22}(x_k) + \\ + c^E_{12}\varepsilon^{(m)}_{33}(x_k) - e_{31}E^{(m)}_3(x_k), \tag{7}$$

$$\sigma^{(m)}_{22}(x_k) = c^E_{12}\varepsilon^{(m)}_{11}(x_k) + c^E_{22}\varepsilon^{(m)}_{22}(x_k) + \\ + c^E_{12}\varepsilon^{(m)}_{33}(x_k) - e_{31}E^{(m)}_3(x_k), \tag{8}$$

$$\sigma^{(m)}_{33}(x_k) = c^E_{12}\varepsilon^{(m)}_{11}(x_k) + c^E_{12}\varepsilon^{(m)}_{22}(x_k) + \\ + c^E_{33}\varepsilon^{(m)}_{33}(x_k) - e_{33}E^{(m)}_3(x_k), \tag{9}$$

where $\sigma^{(m)}_{11}(x_k)$, $\sigma^{(m)}_{22}(x_k)$, $\sigma^{(m)}_{33}(x_k)$ are the axial components of the mechanical stresses in the m-th region of a deformable piezoceramic rod; $m = 1, 2, \ldots, 5$ is the number of the area of the rod (Fig. 1), in which the stress-strain state is determined; when writing relations (7)–(9), material constants of the same value are denoted, as is customary in the mechanics of a deformable solid, by the same symbols. Compression-tension deformations in the m-th area of the rod along the coordinate axes, Ox_1, Ox_2 and Ox_3 (Fig. 1) are denoted by symbols $\varepsilon^{(m)}_{11}(x_k)$, $\varepsilon^{(m)}_{22}(x_k)$ and $\varepsilon^{(m)}_{33}(x_k)$ respectively; $E^{(m)}_3(x_k)$ is an axial component of the electric field strength vector in the m-th region of a deformable piezoceramic rod.

We will consider such a range of frequencies of changing the sign of the potential difference at the output of the generator of electrical signals, in which the wavelength of elastic vibrations of the material particles of the rod is commensurate with its length L. So, according to the definition of a prismatic rod, strong inequalities $\alpha/L \ll 1$ and $b/L \ll 1$ must be fulfilled without fail, then it can be argued that in this frequency range the stress-strain state of the rod does not change within the area of its cross-section.

Suppose that the rod vibrates in a vacuum and does not come into contact with other material objects. In this case, due to Newton's third law the following relations must be fulfilled on its faces:

$$\sigma^{(m)}_{11}(x_k)\Big|_{x_1=0,\,b} = 0, \tag{10}$$

$$\sigma_{22}^{(m)}(x_k)\Big|_{x_2=0}=0,\quad \sigma_{22}^{(m)}(x_k)\Big|_{x_2=L}=0, \tag{11}$$

$$\sigma_{33}^{(m)}(x_k)\Big|_{x_3=0\,,\,\alpha}=0. \tag{12}$$

Conditions (10) and (12) in the considered frequency range are satisfied not only on the surface of the rod, but also at any point inside its volume. This allows us to write relations (7) and (9) in the following form

$$\begin{aligned} c_{11}^E\varepsilon_{11}^{(m)}(x_k)+c_{12}^E\varepsilon_{33}^{(m)}(x_k)&=-c_{12}^E\varepsilon_{22}^{(m)}(x_k)+e_{31}E_3^{(m)}(x_k)\,,\\ c_{12}^E\varepsilon_{11}^{(m)}(x_k)+c_{33}^E\varepsilon_{33}^{(m)}(x_k)&=-c_{12}^E\varepsilon_{22}^{(m)}(x_k)+e_{33}E_3^{(m)}(x_k)\,. \end{aligned} \tag{13}$$

The system of equations (13) determines the components of the deformation tensor $\varepsilon_{11}^{(m)}(x_k)$ and $\varepsilon_{33}^{(m)}(x_k)$:

$$\begin{aligned} \varepsilon_{11}^{(m)}(x_k)&=-\frac{c_{12}^E}{\Delta_0}\left(c_{33}^E-c_{12}^E\right)\varepsilon_{22}^{(m)}(x_k)+\\ &\quad+\frac{1}{\Delta_0}\left(c_{33}^E e_{31}-c_{12}^E e_{33}\right)E_3^{(m)}(x_k)\,,\\ \varepsilon_{33}^{(m)}(x_k)&=-\frac{c_{12}^E}{\Delta_0}\left(c_{11}^E-c_{12}^E\right)\varepsilon_{22}^{(m)}(x_k)+\\ &\quad+\frac{1}{\Delta_0}\left(c_{11}^E e_{33}-c_{12}^E e_{31}\right)E_3^{(m)}(x_k)\,, \end{aligned} \tag{14}$$

where $\Delta_0=c_{11}^E c_{33}^E-\left(c_{12}^E\right)^2$. When writing expressions (14), it was taken into account that the physical state of a deformable piezoelectric remains unchanged within its cross-sectional area.

Substituting expressions (14) into relation (8), we obtain a record of the generalized Hooke's law for a uniaxial stress state of a piezoceramic rod in the low frequency region in the following form

$$\sigma_{22}^{(m)}(x_2)=Y^E\varepsilon_{22}^{(m)}(x_2)-e_{31}^*E_3^{(m)}(x_2)\,, \tag{15}$$

where $Y^E=c_{11}^E-\left(c_{12}^E\right)^2\left(c_{11}^E+c_{33}^E-2c_{12}^E\right)/\Delta_0$ is the Young's modulus of piezoceramics polarized along the axis Ox_3 for the mode of uniaxial compression-tension of the rod along the axis Ox_2; $e_{31}^*=e_{31}+c_{12}^E\left[-e_{31}c_{33}^E-e_{33}c_{11}^E+c_{12}^E\left(e_{31}+e_{33}\right)\right]/\Delta_0$ is the piezoelectric modulus for uniaxial stress state mode of a piezoelectric rod.

The electrical state of a deformable piezoceramic rod is determined by the electric induction vector $\vec{D}^{(m)}(x_k)$, the k-th component of which is generally determined as follows [21]:

$$D_k^{(m)}(x_k)=e_{kij}\varepsilon_{ij}^{(m)}(x_k)+\chi_{kj}^{\varepsilon}E_j^{(m)}(x_k)\,. \tag{16}$$

Since shear deformations in the considered piezoceramic rod are absent by definition, from the general formulation (16) it is possible, taking into account the structure of matrices (5) and (6), to write the following calculation formulas:

$$D_1^{(m)}(x_k)=\chi_{11}^{\varepsilon}E_1^{(m)}(x_k)\,,\quad D_2^{(m)}(x_k)=\chi_{11}^{\varepsilon}E_2^{(m)}(x_k)\,, \tag{17}$$

$$\begin{aligned} D_3^{(m)}(x_k)&=e_{31}\left[\varepsilon_{11}^{(m)}(x_k)+\varepsilon_{22}^{(m)}(x_k)\right]+\\ &\quad+e_{33}\varepsilon_{33}^{(m)}(x_k)+\chi_{33}^{\varepsilon}E_3^{(m)}(x_k)\,. \end{aligned} \tag{18}$$

Since on the edges $x_1=0,\,b$ and $x_2=0,\,L$ the conditions [22]

$$D_1^{(m)}(x_k)\Big|_{x_1=0,\,b}=0,\quad D_2^{(m)}(x_k)\Big|_{x_2=0,\,L}=0$$

must be met, then $E_1^{(m)}(x_k)=0$ and $E_2^{(m)}(x_k)=0$ on the surface and in the volume of the deformable piezoceramic rod. Hence it follows that $D_1^{(m)}(x_k)=D_2^{(m)}(x_k)=0\,\forall\,x_k\in V$, where V is the volume of the prismatic rod.

Thus, the electrical state of a deformable piezoceramic rod is determined by the axial component $D_3^{(m)}(x_2)$ of the electric induction vector, the value of which does not depend on the coordinate x_3. For this reason, the fundamental condition for the absence of free carriers of electricity $div\,\vec{D}^{(m)}(x_k)\equiv\partial D_3^{(m)}(x_k)/\partial x_3=0$ is fulfilled automatically. Substituting expressions (14) into relation (18), we obtain a description of the electric polarization of a piezoceramic rod for the mode of uniaxial stress state:

$$D_3^{(m)}(x_2)=e_{31}^*\varepsilon_{22}^{(m)}(x_2)+\chi_{33}^{\sigma}E_3^{(m)}(x_2)\,, \tag{19}$$

where $\chi_{33}^{\sigma}=\chi_{33}^{\varepsilon}+\left(e_{31}^2c_{33}^E+e_{33}^2c_{11}^E-2e_{31}e_{33}c_{12}^E\right)/\Delta_0$ is the dielectric permittivity for the mode of constancy (equality to zero) of mechanical stresses $\sigma_{11}^{(m)}(x_k)$ and $\sigma_{33}^{(m)}(x_k)$.

In areas 1, 3 and 5, where there is no electroding on the surface $x_3=\alpha$, the electric induction $D_3^{(n)}(x_2)$ ($n=1,3,5$) is equal to zero on the surface $x_3=\alpha$ and, as a consequence, at any point in the volume of the n-th area. Equating to zero the left side of expression (19), we come to the conclusion that in these areas $E_3^{(n)}(x_2)=-e_{31}^*\varepsilon_{22}^{(n)}(x_2)/\chi_{33}^{\varepsilon}$. In this case, the generalized Hooke's law for the n-th region is written in the following form

$$\sigma_{22}^{(n)}(x_2)=Y^D\varepsilon_{22}^{(n)}(x_2)\,, \tag{20}$$

where $Y^D=Y^E+\left(e_{31}^*\right)^2/\chi_{33}^{\sigma}$ is the Young's modulus for the mode of constancy (equality to zero) of electric induction in a deformable piezoelectric.

In order to determine the axial component $E_3^{(k)}(x_2)$ ($k=2,4$) of the electric field strength vector under the electrodes of the primary and secondary electrical circuits of piezoelectric transformer, we write $E_3^{(k)}(x_2)=-\partial\,\Phi^{(k)}(x_2,x_3)/\partial\,x_3$, where $\Phi^{(k)}(x_2,x_3)$ is the scalar potential of the electric field in the k-th region of the deformable piezoceramic rod. Substituting this definition into relation (19) and integrating the result obtained with respect to a variable x_3 in the range from zero to α, we come to the conclusion that

$$D_3^{(k)}(x_2) = e_{31}^* \varepsilon_{22}^{(k)}(x_2) - \chi_{33}^\sigma \frac{U^{(k)}}{\alpha}, \qquad (21)$$

where $U^{(k)}$ is the amplitude value of the electric potential on the surface $x_3 = \alpha$ in the k-th region. It is obvious (Fig. 1) that $U^{(2)} = U_0$, and $U^{(4)} = U_2$. Comparing two physically equivalent definitions of electrical induction $D_3^{(k)}(x_2)$, that is, equating each other the right-hand sides of expressions (19) and (21), we obtain formulas for calculating the amplitude value of the axial component of the electric field strength vector in areas 2 and 4:

$$E_3^{(2)}(x_2) = -U_0/\alpha\,, \quad E_3^{(4)}(x_2) = -U_2/\alpha\,. \qquad (22)$$

Thus, the amplitude values of mechanical stresses in the areas of the rod under the electrodes of the primary and secondary electrical circuits of the transformer are calculated by the following formulas:

$$\begin{aligned} \sigma_{22}^{(2)}(x_2) &= Y^E \varepsilon_{22}^{(2)}(x_2) + e_{31}^* \frac{U_0}{\alpha}, \\ \sigma_{22}^{(4)}(x_2) &= Y^E \varepsilon_{22}^{(4)}(x_2) + e_{31}^* \frac{U_2}{\alpha}. \end{aligned} \qquad (23)$$

Compression-tensile deformations along the axis Ox_2, i.e., the values $\varepsilon_{22}^{(k)}(x_2)$ $(k = 1, 2, 3, 4, 5)$ are determined through the amplitude value of the longitudinal component $u_2^{(k)}(x_2)$ of the displacement vector of material particles of the k-th section of the piezoceramic rod. In this case $\varepsilon_{22}^{(k)}(x_2) = \partial\, u_2^{(k)}(x_2)/\partial\, x_2$. Elastic displacements $u_2^{(k)}(x_2)$ from the equilibrium position of the material particles of the rod satisfy the equation of steady-state harmonic vibrations

$$\frac{\partial\, \sigma_{22}^{(k)}(x_2)}{\partial\, x_2} + \rho_0 \omega^2 u_2^{(k)}(x_2) = 0, \qquad (24)$$

where ρ_0 is the density of piezoceramics.

Substituting the definition of mechanical stresses $\sigma_{22}^{(k)}(x_2)$ on different parts of the piezoceramic rod into equation (24), we write down its general solution in the following form

$$\begin{aligned} u_2^{(1)}(x_2) &= A_1 \sin\gamma x_2 + B\cos\gamma x_2, \\ u_2^{(2)}(x_2) &= A_2 \sin\lambda x_2 + A_3\, \cos\lambda x_2, \\ u_2^{(3)}(x_2) &= A_4 \sin\gamma x_2 + A_5\, \cos\gamma x_2, \\ u_2^{(4)}(x_2) &= A_6 \sin\lambda x_2 + A_7\, \cos\lambda x_2, \\ u_2^{(5)}(x_2) &= A_8 \sin\gamma x_2 + A_9\, \cos\gamma x_2, \end{aligned} \qquad (25)$$

where $A_1 \ldots, A_9$ and B are the constants to be determined; $\gamma = \omega\sqrt{\rho_0/Y^D}$, $\lambda = \omega\sqrt{\rho_0/Y^E}$ are wave numbers of longitudinal vibrations of material particles of piezoelectric ceramics in areas without electrode coating of the surface $x_3 = \alpha$ and on electroded areas of the rod, respectively.

For any method of fixing a piezoceramic rod on the conditional boundaries identified in the process of calculating the regions, the conditions for dynamic and kinematic conjugation of solutions (25) must be satisfied. These conditions are written as follows

$$\begin{aligned} &\sigma_{22}^{(1)}(\ell_1) = \sigma_{22}^{(2)}(\ell_1)\,, \quad \sigma_{22}^{(2)}(\ell_2) = \sigma_{22}^{(3)}(\ell_2)\,, \\ &\sigma_{22}^{(3)}(\ell_3) = \sigma_{22}^{(4)}(\ell_3)\,, \quad \sigma_{22}^{(4)}(\ell_4) = \sigma_{22}^{(5)}(\ell_4)\,; \end{aligned} \qquad (26)$$

$$\begin{aligned} &u_2^{(1)}(\ell_1) = u_2^{(2)}(\ell_1)\,, \quad u_2^{(2)}(\ell_2) = u_2^{(3)}(\ell_2)\,, \\ &u_2^{(3)}(\ell_3) = u_2^{(4)}(\ell_3)\,, \quad u_2^{(4)}(\ell_4) = u_2^{(5)}(\ell_4)\,. \end{aligned} \qquad (27)$$

Conditions at the boundaries $x_2 = 0$ and $x_2 = L$ are determined by the method of fixing the ends of the rod. Suppose that the ends of the rod are free from contacts with other material objects. It follows from this that, as required by Newton's third law, on the surfaces $x_2 = 0$ and $x_2 = L$ the following conditions must be fulfilled

$$\sigma_{22}^{(1)}(0) = 0, \quad \sigma_{22}^{(5)}(L) = 0. \qquad (28)$$

It is easy to calculate that the total number of conditions (26)–(28) exactly corresponds to the number of sought constants.

To complete the construction of a mathematical model of a rod transformer, it is necessary to explicitly define the constants $A_1, \ldots, A_9$ and B. From the first boundary condition (28) it obviously follows that the constant $B = 0$. The remaining nine constants $A_1, \ldots, A_9$ are determined from eight conditions (26), (27) of conjugation of solutions (25) of equation (24) and the second boundary condition (28). Substituting the definitions of stresses $\sigma_{22}^{(k)}(x_2)$ and longitudinal displacements $u_2^{(k)}(x_2)$ into these conditions, we obtain the following system of linear algebraic equations

$$m_{ij} A_j = P_i, \quad i, j = 1, 2, \ldots, 9, \qquad (29)$$

where m_{ij} are frequency dependent coefficients, the numerical values of which are determined as follows: $m_{11} = -Y^D \gamma \sin\gamma\ell_1$; $m_{12} = -Y^E \lambda \cos\lambda\ell_1$; $m_{13} = Y^E \lambda \sin\lambda\ell_1$; $(m_{14} \div m_{19}) = 0$; $m_{21} = 0$; $m_{22} = Y^E \lambda \cos\lambda\ell_2$; $m_{23} = -Y^E \lambda \sin\lambda\ell_2$; $m_{24} = -Y^D \gamma \cos\gamma\ell_2$; $m_{25} = Y^D \gamma \sin\gamma\ell_2$; $(m_{26} \div m_{29}) = 0$; $(m_{31} \div m_{33}) = 0$; $m_{34} = Y^D \gamma \cos\gamma\ell_3$; $m_{35} = -Y^D \gamma \sin\gamma\ell_3$; $m_{36} = -Y^E \lambda \cos\lambda\ell_3$; $m_{37} = Y^E \lambda \sin\lambda\ell_3$; $(m_{38} \div m_{39}) = 0$; $(m_{41} \div m_{45}) = 0$; $m_{46} = Y^E \lambda \cos\lambda\ell_4$; $m_{47} = -Y^E \lambda \sin\lambda\ell_4$; $m_{48} = -Y^D \gamma \cos\gamma\ell_4$; $m_{49} = Y^D \gamma \sin\gamma\ell_4$; $m_{51} = \cos\gamma\ell_1$; $m_{52} = -\sin\lambda\ell_1$; $m_{53} = -\cos\lambda\ell_1$; $(m_{54} \div m_{59}) = 0$; $m_{61} = 0$; $m_{62} = \sin\lambda\ell_2$; $m_{63} = \cos\lambda\ell_2$; $m_{64} = \sin\gamma\ell_2$; $m_{65} = -\cos\gamma\ell_2$; $(m_{66} \div m_{69}) = 0$; $(m_{71} \div m_{73}) = 0$; $m_{74} = \sin\gamma\ell_3$; $m_{75} = \cos\gamma\ell_3$; $m_{76} = -\sin\lambda\ell_3$; $m_{77} = -\cos\lambda\ell_3$; $(m_{78} \div m_{79}) = 0$; $(m_{81} \div m_{85}) = 0$; $m_{86} = \sin\lambda\ell_4$; $m_{87} = \cos\lambda\ell_4$; $m_{88} = -\sin\gamma\ell_4$; $m_{89} = -\cos\gamma\ell_4$; $(m_{91} \div m_{97}) = 0$; $m_{98} = Y^D \gamma \cos\gamma L$;

$m_{99} = -Y^D \gamma \sin \gamma L$;
$P_1 = e_{31}^* U_0/\alpha$; $P_2 = -e_{31}^* U_0/\alpha$; $P_3 = e_{31}^* U_2/\alpha$;
$P_4 = -e_{31}^* U_2/\alpha$; $(P_5 \div P_9) = 0$.

From the design of the system of equations (29) it clearly follows that the stress-strain state of the rod in all its sections without exception depends simultaneously on the amplitude values of the potentials U_0 and U_2, that is, on the potentials on the electrodes of the primary and secondary electrical circuits of the piezoelectric transformer. The latter means that the potentials U_0 and U_2 are related to each other by a linear dependence. This dependence is a formal (mathematical) description of the acoustic (mechanical) connection that exists in a solid piezoceramic rod between wave fields (material particles) in its various sections.

In order to determine the coupling coefficient between the amplitude values of the potentials U_0 and U_2, we construct an expression for calculating the potential U_2.

In accordance with the design diagram shown in Fig. 1, it is necessary to write down that

$$U_2 = Z_2 I_2, \tag{30}$$

where I_2 is the amplitude value of the current in the conductor of the secondary electrical circuit of the piezoelectric transformer. Earlier it was shown [21] that $I_2 = -i\omega Q_2$, where Q_2 is the amplitude value of the electric charge on the electrode of the secondary electrical circuit. It is obvious that

$$Q_2 = b \int_{\ell_3}^{\ell_4} D_3^{(4)}(x_2, \alpha)\, dx_2. \tag{31}$$

Substituting the calculated formula (21) into definition (31), we obtain the following result

$$Q_2 = b\left\{ e_{31}^* \left[u_2^{(4)}(\ell_4) - u_2^{(4)}(\ell_3) \right] - (\ell_4 - \ell_3)\, \chi_{33}^{\sigma} \frac{U_2}{\alpha} \right\} = \\ = b e_{31}^* \left[A_6 (\sin \lambda\ell_4 - \sin \lambda\ell_3) + A_7 (\cos \lambda\ell_4 - \cos \lambda\ell_3) \right] - \\ - C_4^{\sigma} U_2, \tag{32}$$

where $C_4^{\sigma} = b\,(\ell_4 - \ell_3)\, \chi_{33}^{\sigma}/\alpha$ is the dynamic electrical capacitance of the section of the piezoceramic rod under the electrode of the secondary electrical circuit.

The constants A_6 and A_7 are determined from the system of equations (29) as follows:

$$A_6 = -\frac{e_{31}^* U_0}{\alpha} \xi_1 - \frac{e_{31}^* U_2}{\alpha} \xi_2, \quad A_7 = \frac{e_{31}^* U_0}{\alpha} \xi_3 + \frac{e_{31}^* U_2}{\alpha} \xi_4, \tag{33}$$

where $\xi_1 = (\Delta_{61} + \Delta_{62})/\Delta_0$; $\xi_2 = (\Delta_{63} + \Delta_{64})/\Delta_0$; $\xi_3 = (\Delta_{71} + \Delta_{72})/\Delta_0$; $\xi_1 = (\Delta_{73} + \Delta_{74})/\Delta_0$; Δ_{ij} are algebraic additions (determinants of matrices of 8×8 size), which are obtained as a result of deleting the i-th column and j-th row from the matrix of 9×9 size, composed of coefficients at constants A_j in the system of equations (29); Δ_0 is the determinant of the matrix 9×9 of the coefficients at the constants A_j, i.e., the main determinant of the system of equations (29).

Substituting relations (33) into the definition (32) of the amplitude value of the electric charge Q_2, we can write the following

$$Q_2 = C_4^{\sigma} U_2 \Psi_2(\omega) + C_4^{\sigma} U_0 \Psi_0(\omega),$$

where

$$\Psi_2(\omega) = \frac{(e_{31}^*)^2}{(\ell_4 - \ell_3)\, \chi_{33}^{\sigma}} \times \\ \times \left[\xi_4 (\cos \lambda\ell_4 - \cos \lambda\ell_3) - \xi_3 (\sin \lambda\ell_4 - \sin \lambda\ell_3) \right] - 1;$$

$$\Psi_0(\omega) = \frac{(e_{31}^*)^2}{(\ell_4 - \ell_3)\, \chi_{33}^{\sigma}} \times \\ \times \left[\xi_2 (\cos \lambda\ell_4 - \cos \lambda\ell_3) - \xi_1 (\sin \lambda\ell_4 - \sin \lambda\ell_3) \right].$$

Since $I_2 = -i\omega C_4^{\sigma} Z_2 U_2 \Psi_2(\omega) - i\omega C_4^{\sigma} Z_2 U_0 \Psi_0(\omega)$, then after substituting the amplitude value of the current into the definition (30), we obtain the following equality

$$U_2 \left[1 + i\omega C_4^{\sigma} Z_2 \Psi_2(\omega) \right] = -i\omega C_4^{\sigma} Z_2 U_0 \Psi_0(\omega),$$

whence it follows that

$$U_2 = -\Xi_0(\omega)\, U_0, \tag{34}$$

where $\Xi_0(\omega)$ is an acoustic feedback coefficient, the numerical values of which are calculated by the formula

$$\Xi_0(\omega) = \frac{i\omega C_4^{\sigma} Z_2 \Psi_0(\omega)}{1 + i\omega C_4^{\sigma} Z_2 \Psi_2(\omega)}. \tag{35}$$

It is obvious (Fig. 1) that the amplitude value of the potential is determined as follows

$$U_0 = \frac{U_1 Z_{el}^{(1)}(\omega)}{Z_1 + Z_{el}^{(1)}(\omega)}, \tag{36}$$

where $Z_{el}^{(1)}(\omega)$ is an electrical impedance of the piezoceramic rod section under the electrode of the primary electrical circuit of the piezoelectric transformer.

Let us determine the electrical impedance $Z_{el}^{(1)}(\omega)$. This operation actually completes the construction of a mathematical model of a rod piezoelectric transformer.

From the well-known Ohm's law for a section of an electrical circuit, it follows that

$$Z_{el}^{(1)}(\omega) = \frac{U_0}{I_1}, \tag{37}$$

where I_1 is the amplitude value of the electric current in the conductor of the primary electrical circuit. Since $I_1 = -i\omega Q_1$, and the electric charge Q_1 is determined through electric induction $D_3^{(2)}(x_2, \alpha)$, then

$$I_1 = -i\omega b e_{31}^* \left[u_2^{(2)}(\ell_2) - u_2^{(2)}(\ell_1) \right] + i\omega C_2^{\sigma} U_0, \tag{38}$$

where $C_2^{\sigma} = b\,(\ell_2 - \ell_1)\, \chi_{33}^{\sigma}/\alpha$ is the dynamic electrical capacitance of the section of the piezoceramic rod under the electrode of the primary electrical circuit of the piezoelectric transformer.

The coefficients A_2 and A_3, which determine the longitudinal displacements of the material particles of the rod in the sections $x_2 = \ell_1$ and $x_2 = \ell_2$, are calculated by the following formulas

$$A_2 = -\frac{e_{31}^* U_0}{\alpha}\zeta_1 - \frac{e_{31}^* U_2}{\alpha}\zeta_2, \quad A_3 = \frac{e_{31}^* U_0}{\alpha}\zeta_3 + \frac{e_{31}^* U_2}{\alpha}\zeta_4, \tag{39}$$

where $\zeta_1 = (\Delta_{21} + \Delta_{22})/\Delta_0$; $\zeta_2 = (\Delta_{23} + \Delta_{24})/\Delta_0$; $\zeta_3 = (\Delta_{31} + \Delta_{32})/\Delta_0$; $\zeta_4 = (\Delta_{33} + \Delta_{34})/\Delta_0$.

Substituting the definitions (39) of the constants A_2 and A_3 into the formula for calculating the displacement $u_2^{(2)}(x_2)$ of the material particles of the rod, and the obtained results into relation (38), we obtain the following result

$$I_1 = -i\omega C_2^{\sigma} U_0 \Psi_1(\omega), \tag{40}$$

where

$$\Psi_1(\omega) = \frac{(e_{31}^*)^2}{(\ell_2 - \ell_1)\chi_{33}^{\sigma}} \{-[\zeta_1 - \Xi_0(\omega)\zeta_2](\sin\lambda\ell_2 - \sin\lambda\ell_1) + [\zeta_3 - \Xi_0(\omega)\zeta_4](\cos\lambda\ell_2 - \cos\lambda\ell_1)\} - 1.$$

Taking into account the definition (40) of the amplitude value of the current in the conductor of the primary electrical circuit, from the relation (37) we find the electrical impedance

$$Z_{el}^{(1)}(\omega) = \frac{1}{-i\omega C_2^{\sigma}\Psi_1(\omega)},$$

after which we obtain a calculation formula for the amplitude value of the potential U_0 from the relation (36)

$$U_0 = \frac{U_1}{1 - i\omega C_2^{\sigma} Z_1 \Psi_1(\omega)}. \tag{41}$$

Substituting expression (41) into relation (34), we find the calculation formula for the potential U_2, after which we can write expression (1) to calculate the transformation ratio $K(\omega, \Pi)$ in the following form

$$K(\omega, \Pi) = \frac{U_2}{U_1} = -\frac{\Xi_0(\omega)}{1 - i\omega C_2^{\sigma} Z_1 \Psi_1(\omega)}. \tag{42}$$

The analytical structure (42) is a mathematical model of a rod piezoelectric transformer, the principle of operation of which involves the use of longitudinal vibrations in a prismatic rod, the design diagram of which is shown in Fig. 1.

4 Discussion and experimental confirmation of simulation results

For practical confirmation of the obtained simulation results, a practical experiment was implemented [23]. It was carried out at the research stand [24] in the frequency range 0-80 kHz and was used to confirm the results of mathematical modelling with the above sequence. The experiment was carried out under the condition of observing the total measurement error, which did not exceed 8.5%, Fig. 2 (dashed line).

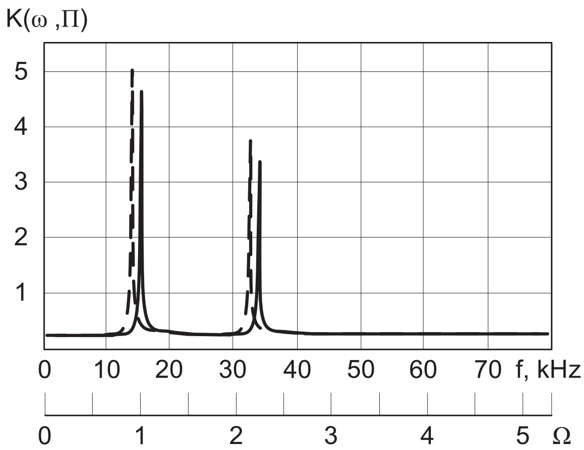

Fig. 2. Calculated (solid line) and experimentally obtained (dashed line) curves of the frequency dependence of the modulus of the transformation ratio of a piezoceramic transducer

The results of mathematical modelling (the transformation ratio $K(\omega, \Pi)$) of rod transducer with a square cross-section made of PZT type piezoelectric ceramics were acquired according to the expression (42) and are presented in the form of dependences (solid curves) shown in Fig. 2. The following parameter values were taken for calculations: $c_{11}^E = 112\,GPa$; $c_{12}^E = 62\,GPa$; $c_{33}^E = 106\,GPa$; $\rho_0 = 7400\ {}^{kg}/_{m^3}$; $e_{33} = 18\ {}^{C}/_{m^2}$; $e_{31} = -7\ {}^{C}/_{m^2}$; $\chi_{33}^{\varepsilon} = 8,85 \cdot 10^{-9}\ {}^{F}/_{m}$; $Q_0 = 80$; $\alpha = \beta = 9\,mm$; $L = 90\,mm$. The calculations were performed at the frequencies of the first two electromechanical resonances.

When comparing the results of mathematical modeling and the results of experimental measurements (Fig. 2), one can see that the calculated results agree with the actual values of the amplitude-frequency characteristic. At the same time, the discrepancy between the results of mathematical modeling and the experimentally obtained data at the same values of the operating frequency in the range up to 50 kHz is 7.8-8.5%, and in the range of operating frequencies of 50-80 kHz it does not exceed 5.1%).

The obtained calculated results are extremely important from a practical point of view, since for the overwhelming majority of real situations the use of piezoelectric transducers becomes impossible, due to the impossibility of obtaining a posteriori information on the actual values of the transformation ratio modulus in the process of experimental use of such transducers. Therefore, the development of a mathematical model of rod-type piezoelectric transducers, the implementation of which can be automated with the involvement of modern microprocessor tools and software, will make it possible to quickly and with sufficient accuracy build the dependence of the transformation ratio on the cyclic frequency and operating parameters of the piezoelectric transducer, which will improve the development process of such transducers. and will also reduce the time and cost of the technological process of their manufacture.

Thus, as a result of the comparison of the calculated and experimental curves of the frequency dependence of the modulus of the transformation ratio, the possibility of using the mathematical model developed by the authors for calculating the main operational characteristics of piezoceramic rod-type transducers in the process of designing acoustoelectronic devices has been proved.

Conclusions

The sequence of constructing a mathematical model of piezoelectric transducers of a rod type (rod piezoelectric transformer) is presented, the result of which is finding a complex-valued function $K(\omega, \Pi)$ on the cyclic frequency and a complex of parameters (geometric, physical-mechanical, electrical) of a piezoelectric transformer.

Analytical dependences was obtained that made it possible to determine the electrical impedance and the amplitude value of the potential that arise in the electrical circuit of a piezoelectric rod-type transducer with a square cross-section from piezoelectric ceramics of the PZT type.

A comparison of the calculated and experimental values of the frequency dependence of the module of the transformation ratio of the piezoceramic transducer was carried out, which showed a high convergence between these data (the discrepancy between the results of mathematical modelling and the experimentally obtained data at the same values of the operating frequency in the range up to 80 *kHz* did not exceed 8.5%).

The above results were obtained as part of the scientific and technical (experimental) project "Development of a highly efficient mobile ultrasonic system for intensifying the extraction process in the manufacture of concentrated functional drinks", which is carried out at the expense of the general fund of the state budget (state registration number: 0121U109660, registration date: 12.03.2021).

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Математичне моделювання п'єзоелектричних перетворювачів стрижньового типу для пристроїв акустоелектроніки

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Робота присвячена особливостям побудови та дослідження математичних моделей п'єзоелектричних перетворювачів стрижньового типу, які знаходять широке застосування в різноманітних пристроях акустоелектроніки (гідроакустичних засобів виявлення цілей, ультразвукового неруйнівного контролю, медичної діагностики, тощо). На відміну від існуючих математичних моделей п'єзоелектричних перетворювачів (на основі амплітудно-фазових залежностей, резонансних п'єзоелектричних перетворювачів, еквівалентних схем та інших), запропонована математична модель дозволяє встановити залежність, що є математичним описом акустичного зв'язку, який існує в суцільному п'єзокерамічному стрижні між хвильовими полями на його різних ділянках.

Наведено алгоритм розрахунку математичної моделі п'єзоелектричних перетворювачів стрижньового типу, що базується на визначені коефіцієнта трансформації, який виникає при зворотному п'єзоелектричному ефекті. Отримані аналітичні залежності, що дозволяють визначати електричний імпеданс та амплітудне значення потенціалу в електричному колі п'єзоелектричного перетворювача. Показано, що ці залежності лежать в основі виразу для визначення коефіцієнта трансформації $K(\omega, \Pi)$, який є математичною моделлю стрижневого п'єзоелектричного трансформатора. При цьому, принцип дії такого п'єзотрансформатора передбачає використання поздовжніх коливань в призматичному стрижні.

Наведені результати проведеного математичного моделювання на прикладі стрижневого перетворювача з квадратним поперечним перерізом із п'єзоелектричної кераміки типу ЦТС. Проведені порівняння розрахованих та експериментально отриманих значень частотної залежності модуля коефіцієнта трансформації п'єзокерамічного перетворювача показали високу збіжність між ними (розбіжність між результатами математичного моделювання та експериментально отриманими даними для однакового значення робочої частоти не перевищує 8,5%).

Ключові слова: п'єзоелектричний перетворювач; акустоелектроніка; математична модель; генератор електричних сигналів

Математическое моделирование пьезоэлектрических преобразователей стержневого типа для устройств акустоэлектроники

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Работа посвящена особенностям построения и исследования математических моделей пьезоэлектрических преобразователей стержневого типа, которые находят широкое применение в различных устройствах акустоэлектроники (гидроакустических средств обнаружения целей, ультразвукового неразрушающего контроля, медицинской диагностики и т.д.). В отличие от существующих математических моделей пьезоэлектрических преобразователей (основанных на амплитудно-фазовых зависимостях, резонансных пьезоэлектрических преобразователях, эквивалентных схемах и т.п.), предложенная математическая модель позволяет установить зависимость, являющуюся математическим описанием акустической связи, которая существует в твердом пьезокерамическом стержне между волновыми полями на его различных участках.

Приведен алгоритм расчета математической модели пьезоэлектрических преобразователей стержневого типа, основанный на определении коэффициента трансформации, который возникает при обратном пьезоэлектрическом эффекте. Получены аналитические зависимости, позволяющие определять электрический импеданс и амплитудное значение потенциала в электрической цепи пьезоэлектрического преобразователя. Показано, что эти зависимости лежат в основе выражения для определения коэффициента трансформации K(ω, П), который является математической моделью стержневого пьезоэлектрического трансформатора. При этом, принцип действия такого пьезотрансформатора предусматривает использование продольных колебаний в призматическом стержне.

Приведены результаты проведенного математического моделирования на примере стержневого преобразователя с квадратным поперечным сечением из пьезоэлектрической керамики типа ЦТС. Проведены сравнения рассчитанных и экспериментально полученных значений частотной зависимости модуля коэффициента трансформации пьезокерамического преобразователя показали высокую сходимость между ними (расхождение между результатами математического моделирования и экспериментально полученными данными для одинакового значения рабочей частоты не превышает 8,5%).

Ключевые слова: пьезоэлектрический преобразователь; акустоэлектроника; математическая модель; генератор электрических сигналов