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Scattering of Electromagnetic Waves by Frequency-Detuned Systems of Dielectric Resonators

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The general problem of scattering on a system of frequency-detuned coupled Dielectric Resonators (DRs) located in one or several transmission lines is considered. The field describing the natural oscillations of the system of detuned DRs is decomposed over the field of partial resonators. A system of equations is derived, the solution of which allows one to determine the frequencies and amplitudes of the system's natural oscillations. It is shown that the resulting system of equations, by means of algebraic transformations, can be reduced to the problem of determining the eigenfunctions and eigenvalues of a finite-dimensional operator, determined through the elements of the coupling operator of frequency-detuned DRs. The limitations of the proposed calculation method are noted. The solution to the scattering problem is expanded in terms of the natural oscillation field of the system of detuned DRs. A system of linear equations for the amplitudes of forced oscillations is obtained. It is shown that in the particular case of identical resonator frequencies, the found system exactly coincides with the equations obtained previously for various types of DR. General solutions are found for the scattered field on frequency-detuned resonators located in different transmission lines. Several examples are given of calculating the frequency dependences of the scattering matrix for the most interesting structures consisting of coupled dielectric resonators detuned in natural oscillation frequencies. The frequency characteristics of the scattering of two bandstop filters with different stop bands, made on detuned DRs in the same transmission line, are calculated. The frequency dependences of the scattering matrix on two bandpass filters located parallel between regular lines are calculated. The scattering characteristics of bandpass filters of complex designs containing various DRs are calculated: a bandpass filter built on a system of coupled DRs in a transmission line break and several DRs in a regular line, as well as an elliptical bandpass filter. The capabilities of the proposed method are demonstrated by the example of optimizing the scattering characteristics of a demultiplexer built on the basis of two bandpass filters with different passband frequencies. The proposed method also makes it possible to calculate the scattering characteristics of bandpass and bandstop filters with several operating frequency bands used in modern communication systems.

Keywords: scattering; detuned dielectric resonator; scattering matrix; bandpass filter; bandstop filter; elliptical filter; demultiplexer

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Introduction

Today, systems of coupled dielectric resonators (DRs) are used in a variety of devices – filters, antennas, multiplexers from the microwave to visible wavelength ranges [1–31]. As a rule, such devices are made of DRs of the equal size, consisting of the same material. At once, when designing filters with more complex frequency responses [8], as well as when optimizing multiplexers [13, 23, 24], multi-band filters [15–22]; multi-band [25–28] and fractal antennas [29–31], there is a need to describe the frequency characteristics of the scattering of electromagnetic waves in the transmission line on systems contained various coupled resonators with different natural frequencies. A correct analytical solution of this problem has not been found.

1 Statement of the problem

The purpose of this article is to construct an electromagnetic theory of scattering on systems of coupled dielectric resonators with different natural frequencies, located in the transmission line. The proposed solution is based on the consequent application of perturbation theory for describing the coupled oscillations of various DRs with use of obtained eigenfunctions to solving the scattering problem. The correctness of the constructed theory is confirmed by numerical calculations of scattering characteristics on complex systems of bandpass and bandstop filters, as well as multiplexers with different frequency characteristics. In the case of identical resonant frequencies of partial resonators, the constructed theory coincides with those constructed earlier [32,33].

2 Main proposition of scattering theory on detuned DRs

To construct the scattering theory, we use field expansions in terms of coupled oscillations of the DR system [32], so we will first consider the natural oscillations of a system consisting of different DR (Fig. 1). Let us assume that we know the field of natural oscillations of each of the resonators $(\mathbf{e}_s, \mathbf{h}_s)$, as well as the frequency of its natural oscillations in the line: $\tilde{\omega}_s = \omega_s + i\omega_s^{''}$. Let us also assume that each of the resonators is made of a lossless dielectric, i.e. its dielectric constant is purely real: $\tilde{\varepsilon}_s = \varepsilon_s^{'} = \varepsilon_s$ $(s = 1, 2, \dots, N)$.

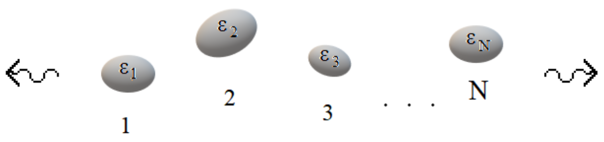

Fig. 1. The system of the coupling DRs

Let us further denote by $(\mathbf{e}, \mathbf{h})$ – the field, and by $\tilde{\omega} = \omega + i\omega''$ – the complex frequency of coupled oscillations of the resonator system.

We make assumption that all DR field amplitudes are vary with frequency by known to us law $\alpha_s(\omega)$:

$$\begin{bmatrix}\mathbf{e}_s(\omega,\mathbf{r})\\ \mathbf{h}_s(\omega,\mathbf{r})\end{bmatrix} \simeq \alpha_s(\omega)\begin{bmatrix}\mathbf{e}_s(\mathbf{r})\\ \mathbf{h}_s(\mathbf{r})\end{bmatrix}. \tag{1}$$

At that we supposed ω_s realized maximum $\alpha_s(\omega)$ in which:

$$\alpha_s(\omega_s) = 1; \quad \text{and} \quad \lim_{|\omega-\omega_s|\to\infty} \alpha_s(\omega) = 0. \tag{2}$$

The solution of eigenoscillation problem of the N-DR system $(\mathbf{e}(\omega,\mathbf{r}), \mathbf{h}(\omega,\mathbf{r}))$ also can be represented as a superposition of fields of the isolated resonators $(\mathbf{e}_s(\omega,\mathbf{r}), \mathbf{h}_s(\omega,\mathbf{r}))$, $(s = 1, 2, \dots, N)$:

$$\begin{aligned}\mathbf{e}(\omega,\mathbf{r}) &= \sum_{s=1}^{N} b_s(\omega)\mathbf{e}_s(\omega,\mathbf{r});\\ \mathbf{h}(\omega,\mathbf{r}) &= \sum_{s=1}^{N} b_s(\omega)\mathbf{h}_s(\omega,\mathbf{r}).\end{aligned} \tag{3}$$

Here $b_s(\omega)$ is the unknown complex amplitude of the s-th resonator mode.

Let us show that despite the fact that the field of each of the partial resonators (1) is a function of frequency, the natural frequencies and distributions of the fields of coupled oscillations of the resonator system, as one would expect, remain constant.

We defined the expansion coefficients of the DR field (1) on the propagation wave field of the line $(\mathbf{E}_t^{\pm}, \mathbf{H}_t^{\pm})$ via the DR surface integrals:

$$\mathrm{c}_{\mathrm{t}}^{\mathrm{s}\pm}(\omega) = -\frac{1}{2}\oint_{\mathrm{s}_R}\Big\{\big[\mathbf{e}_s^1(\omega),\mathbf{n}_R\big]\big(\mathbf{H}_{\mathrm{t}}^{\pm}\big)^* + \big[\mathbf{n}_R,\mathbf{h}_{\mathrm{s}}^1(\omega)\big]\big(\mathbf{E}_{\mathrm{t}}^{\pm}\big)^*\Big\}\mathrm{ds}. \tag{4}$$

And for the non propagating waves:

$$\begin{pmatrix}\mathrm{c}_{\mathrm{t}}^{s\pm}(\omega)\\ \mathrm{d}_{\mathrm{t}}^{s\pm}(\omega)\end{pmatrix} = \\ = (\mp)\frac{\mathrm{i}}{2}\oint_{\mathrm{s}_R}\Big\{\big[\mathbf{e}_s^1(\omega),\mathbf{n}_R\big]\big(H_{\mathrm{t}}^{\mp}\big)^* + \big[\mathbf{n}_R,\mathbf{h}_{\mathrm{s}}^1(\omega)\big]\big(E_{\mathrm{t}}^{\mp}\big)^*\Big\}\mathrm{ds}. \tag{5}$$

Here, $\mathbf{n}_R$ – normal to the s-th resonator surface; $(\mathbf{e}_s^1, \mathbf{h}_s^1)$ – field of s-th resonator in the dielectric. For compactness, the dependence on spatial coordinates $\mathbf{r}$ were omitted.

Then, using (1), we obtained:

$$c_t^{s\pm}(\omega) = \alpha_s(\omega)c_t^{s\pm}; \quad d_t^{s\pm}(\omega) = \alpha_s(\omega)d_t^{s\pm}, \tag{6}$$

where $c_t^{s\pm}$; $d_t^{s\pm}$ are known expansion coefficients at the resonance frequency ω_s for the propagating and non propagating waveguide waves, respectively.

Here we neglected the dependence of the longitudinal wave number of the transmission line on frequency $\Gamma = \Gamma(\omega)$. For simplicity, we will assume that the set of natural oscillation frequencies of partial resonators occupies a not too wide band relative to the average frequency ω_0: $|\omega - \omega_0| \ll \omega_0$ then we can put:

$$\Gamma = \Gamma(\omega) = \Gamma(\omega_0) + \Gamma(\omega_0)'(\omega - \omega_0) + \dots \approx \Gamma(\omega_0).$$

The energy, stored inside s-th DR at frequency ω we define as:

$$w_s(\omega) = 1/4\int_{v_s}\Big[\varepsilon_s\big|\mathbf{e}_s^1(\omega)\big|^2 + \mu_0\big|\mathbf{h}_s^1(\omega)\big|^2\Big]dv,$$

$(s = 1, 2, \dots, N)$. From (1) also obtained:

$$w_s(\omega) = |\alpha_s(\omega)|^2 w_s, \tag{7}$$

where w_s is the energy, stored s-th DR at frequency ω_s.

Mutual coupling coefficients of the s-th and the t-th DRs on the damped and expanding waves at frequency ω are defined as:

$$\begin{aligned}k_{st}(\omega) &= \frac{-1}{\omega_t w_t(\omega)}\times\\ &\times\sum_{n=1}^{\infty}\big[c_n^s(\omega)c_n^t(\omega)^* e^{-\Gamma\Delta z_{st}} - d_n^s(\omega)d_n^t(\omega)^* e^{-\Gamma\Delta z_{st}}\big],\\ \tilde{k}_{st}(\omega) &= \frac{1}{\omega_t w_t(\omega)}c_e^s(\omega)c_e^t(\omega)^* e^{-i\Gamma\Delta z_{st}}.\end{aligned} \tag{8}$$

From (6), (7) obtained:

$$k_{st}(\omega) = \frac{\alpha_s(\omega)}{\alpha_t(\omega)} k_{st}; \quad \tilde{k}_{st}(\omega) = \frac{\alpha_s(\omega)}{\alpha_t(\omega)} \tilde{k}_{st}, \tag{9}$$

where, similarly k_{st}; $\tilde{k}_{st}$ is coupling coefficients, defined formally through fields of the s-th and t-th DR.

Let us consider sequentially bilinear combinations that include the desired field of the system of coupled resonators $(\mathbf{e},\, \mathbf{h})$ (3), as well as the fields of partial DR $(\mathbf{e}_n, \mathbf{h}_n)$ (1):

$$\begin{gathered}\operatorname{div}\{[\mathbf{h}_{\mathrm{n}}^*, \mathbf{e}] + [\mathbf{h},\, \mathbf{e}_{\mathrm{n}}^*]\} = \mathrm{i}\,(\omega - \omega_0)\,[\varepsilon_1 \mathbf{e}\cdot\mathbf{e}_{\mathrm{n}}^* + \mu_0 \mathbf{h}\cdot\mathbf{h}_{\mathrm{n}}^*] - \\ -(\omega'' + \omega_n'')\,[\varepsilon_1 \mathbf{e}\cdot\mathbf{e}_{\mathrm{n}}^* + \mu_0 \mathbf{h}\cdot\mathbf{h}_{\mathrm{n}}^*]\,,\end{gathered} \tag{10}$$

and in addition, the fields of each of the partial resonators:

$$\operatorname{div}\{[\mathbf{h}_{\mathrm{n}}^*, \mathbf{e}_n] + [\mathbf{h}_n,\, \mathbf{e}_{\mathrm{n}}^*]\} = -2\omega_n''\left[\varepsilon_1 |\mathbf{e}_n|^2 + \mu_0 |\mathbf{h}_n|^2\right]. \tag{11}$$

Substituting (3) into (10) and subtracting from it (11), multiplied by $b_n(\omega)$, after integration over the volume V_n of the n-th partial resonator, we find taking into account:

$$\begin{gathered}\left|\int\limits_{V_t} \left(\mathbf{e}_t^1, \mathbf{e}_s^0(\omega)^*\right)\, \mathrm{dv}\right| << \int\limits_{V_t} \left|\mathbf{e}_t^1\right|^2\, \mathrm{dv}; \\ \left|\int\limits_{V_t} \left(\mathbf{h}_t^1, \mathbf{h}_s^0(\omega)^*\right)\, \mathrm{dv}\right| << \int\limits_{V_t} \left|\mathbf{h}_t^1\right|^2\, \mathrm{dv}.\end{gathered} \tag{12}$$

$$\sum_{s\neq t}^{N} \alpha_s(\omega)\kappa_{st} b_s(\omega) + \alpha_t(\omega)(i\tilde{k}_t - \lambda_t) b_t(\omega) = 0, \tag{13}$$

where

$$\lambda_t = 2\cdot\left(\frac{\tilde{\omega} - \omega_t'}{\omega_t'}\right), \quad (t = 1, 2, \ldots, N); \tag{14}$$

$\kappa_{st} = k_{st} + i\tilde{k}_{st}$; ω_t' is the real part of frequency of t-th DR mode.

Main proposition. Effective amplitudes b_s and frequencies $\tilde{\omega}$ of coupled oscillations do not depend on frequency functions $\alpha_s(\omega)$.

Equating the determinant of the system of equations (13) to zero, we find that the functions $\alpha_s(\omega)$ are not included in the characteristic equation at all. In addition, by replacing the variables, $\alpha_s(\omega)b_s(\omega) = b_s$ we come to the conclusion that the new amplitudes b_s are also independent of frequency, since they satisfy a system of frequency-independent equations:

$$\sum_{s\neq t}^{N} \kappa_{st} b_s + (i\tilde{k}_t - \lambda_t) b_t = 0, \quad (t = 1, 2, \ldots, N). \tag{15}$$

System of equations (15) allows one to calculate the frequencies and fields of coupled oscillations of a system of detuned DRs. It differs from the system of equations [33] found earlier for resonators with the same frequencies of natural oscillations, values of the functions λ_t (14). At that, coupling coefficients in (9) are formally defined for different natural frequencies of partial resonators.

Functions λ_t are no longer eigenvalues of the coupling operator $\mathrm{K} = \|\kappa_{st}\|$. However, we can algebraically transform the system of equations (15) so that it again formally becomes an eigenvalue problem for the new coupling operator $\widehat{\mathrm{K}} = \|\widehat{\kappa}_{st}\|$, where

$$\begin{gathered}\widehat{\kappa}_{st} = \frac{\omega_t'}{\omega_0}\kappa_{st}; \qquad \widehat{\kappa}_t = i\frac{\omega_t'}{\omega_0}\tilde{k}_t - 2\frac{\omega_0 - \omega_t'}{\omega_0}; \\ \lambda = 2\cdot\left(\frac{\tilde{\omega} - \omega_0}{\omega_0}\right).\end{gathered} \tag{16}$$

Here ω_0 is an arbitrary frequency, the value of which we will take as the average of the frequencies of partial resonators.

Thus, to determine the amplitudes and frequencies of coupled oscillations of frequency detuned DRs, we need to solve the problem of eigenvalues of the new operator $\widehat{\mathrm{K}}$:

$$\sum_{s\neq t}^{N} \widehat{\kappa}_{st} b_s + (\widehat{\kappa}_t - \lambda) b_t = 0, \quad (t = 1, 2, \ldots, N), \tag{17}$$

where for each eigenvalue $\lambda = \lambda^t$ the field of coupled oscillations takes the simple form:

$$\mathbf{e}^t(\mathbf{r}) = \sum_{s=1}^{N} b_s^t \mathbf{e}_s(\mathbf{r}); \qquad \mathbf{h}^t(\mathbf{r}) = \sum_{s=1}^{N} b_s^t \mathbf{h}_s(\mathbf{r}). \tag{18}$$

The real field of the resonator system, of course, has the form (3).

3 System of equations for the amplitudes of forced oscillations of the detuned DRs

Let us construct a solution to the problem of the wave scattering on a system of coupled detuned DRs, based on the perturbation theory of Maxwell's equations, using expansions (3), and the conclusions of the previous section. Let us assume that a wave $\left(\mathbf{E}_l^+, \mathbf{H}_l^+\right)$ falls on a system of different frequency detuned resonators located in a transmission line. Now we will assume that each of the resonators is made of a lossy dielectric: $\tilde{\varepsilon}_s = \varepsilon_s' - i\varepsilon_s''$ $(s = 1, 2, \ldots, N)$, wherein $\varepsilon_s'' << \varepsilon_s'$.

We present the solution to the scattering problem in the form:

$$\begin{gathered}\mathbf{E}(\omega) \approx \mathbf{E}_l^+ + \sum_{s=1}^{N} a^s \mathbf{e}^s(\omega); \\ \mathbf{H}(\omega) \approx \mathbf{H}_l^+ + \sum_{s=1}^{N} a^s \mathbf{h}^s(\omega),\end{gathered} \tag{19}$$

where $(\mathbf{e}^s(\omega),\ \mathbf{h}^s(\omega))$ – field of coupled oscillations of a resonator system.

From (3):

$$\mathbf{e}^s(\omega) = \sum_{t=1}^{N} b_t^s(\omega)\mathbf{e}_t(\omega); \qquad \mathbf{h}^s(\omega) = \sum_{t=1}^{N} b_t^s(\omega)\mathbf{h}_t(\omega). \tag{20}$$

Using expressions similar to (10), (11) written for the scattered field, transmission line field, and also the fields of partial resonators, we arrive at the equation for amplitudes:

$$\sum_{s=1}^{N} a^s \alpha_t(\omega) b_t^s(\omega) Q_{st}(\omega) = -Q_t^D \left(c_t^+\right)^* / \omega_t w_t,$$

where

$$Q_{st}(\omega) = \omega/\omega_t + 2iQ_t^D(\omega/\omega_t - 1 - \lambda^s/2); \tag{21}$$

$Q_t^D = \omega_t w_t / P_t^D$ – loss quality factor in the dielectric of the t-th DR; $P_t^D = \frac{\omega_t}{2}\varepsilon_t'' \int\limits_{V_t} |\mathbf{e}_t|^2 dv$) determines the dielectric power loss of the t-th resonator.

Taking into account the previously found relations $\alpha_s(\omega) b_s(\omega) = b_s$, we come to the conclusion that the found equation exactly coincides with [33] in which ω_0 it is replaced by ω_t

$$\sum_{s=1}^{N} a^s b_t^s Q_{st}(\omega) = -Q_t^D \left(c_t^+\right)^* / \omega_t w_t. \tag{22}$$

The transmission T and the reflection coefficient R of the detuned DR system in the transmission line can be also obtained in the form:

$$T = T_0 + \sum_{u=1}^{N} \left(\sum_{s=1}^{N} b_s^u c_s^+\right) a_u = T_0 - \frac{1}{B(\omega)} \sum_{s=1}^{N} B_s^+(\omega);$$
$$R = R_0 + \sum_{u=1}^{N} \left(\sum_{s=1}^{N} b_s^u c_s^-\right) a_u = R_0 - \frac{1}{B(\omega)} \sum_{s=1}^{N} B_s^-(\omega). \tag{23}$$

Here T_0, R_0 are the transmission and reflection coefficients of the transmission line without DRs;

$$B_s^{\pm}(\omega) = \det \begin{bmatrix} b_1^1 Q_{11}(\omega) & b_1^2 Q_{21}(\omega) & \dots & Q_1^D \sum_{u=1}^{N} b_u^s \tilde{k}_{u1}^{\pm+} & \dots & b_1^N Q_{N1}(\omega) \\ b_2^1 Q_{12}(\omega) & b_2^2 Q_{22}(\omega) & \dots & Q_2^D \sum_{u=1}^{N} b_u^s \tilde{k}_{u2}^{\pm+} & \dots & b_2^N Q_{N2}(\omega) \\ . & . & \dots & . & \dots & . \\ b_N^1 Q_{1N}(\omega) & b_N^2 Q_{2N}(\omega) & \dots & Q_N^D \sum_{u=1}^{N} b_u^s \tilde{k}_{uN}^{\pm+} & \dots & b_N^N Q_{NN}(\omega) \end{bmatrix},$$
$$B(\omega) = \det \begin{bmatrix} b_1^1 Q_{11}(\omega) & b_1^2 Q_{21}(\omega) & \dots & b_1^N Q_{N1}(\omega) \\ b_2^1 Q_{12}(\omega) & b_2^2 Q_{22}(\omega) & \dots & b_2^N Q_{N2}(\omega) \\ . & . & \dots & . \\ b_N^1 Q_{1N}(\omega) & b_N^2 Q_{2N}(\omega) & \dots & b_N^N Q_{NN}(\omega) \end{bmatrix} \tag{24}$$

for bandstop filters and

$$B_s^{+}(\omega) = \det \begin{bmatrix} b_1^1 Q_{11}(\omega) & b_1^2 Q_{21}(\omega) & \dots & Q_1^D b_N^s \tilde{k}_{N1}^{++} & \dots & b_1^N Q_{N1}(\omega) \\ b_2^1 Q_{12}(\omega) & b_2^2 Q_{22}(\omega) & \dots & 0 & \dots & b_2^N Q_{N2}(\omega) \\ . & . & \dots & . & \dots & . \\ b_N^1 Q_{1N}(\omega) & b_N^2 Q_{2N}(\omega) & \dots & 0 & \dots & b_N^N Q_{NN}(\omega) \end{bmatrix},$$
$$B_s^{-}(\omega) = \det \begin{bmatrix} b_1^1 Q_{11}(\omega) & b_1^2 Q_{21}(\omega) & \dots & Q_1^D b_1^s \tilde{k}_{11}^{-+} & \dots & b_1^N Q_{N1}(\omega) \\ b_2^1 Q_{12}(\omega) & b_2^2 Q_{22}(\omega) & \dots & 0 & \dots & b_2^N Q_{N2}(\omega) \\ . & . & \dots & . & \dots & . \\ b_N^1 Q_{1N}(\omega) & b_N^2 Q_{2N}(\omega) & \dots & 0 & \dots & b_N^N Q_{NN}(\omega) \end{bmatrix} \tag{25}$$

for the band pass filters. Here

$$\tilde{k}_{sn}^{++} = \left(c_s^+ c_n^{+*}\right)/(\omega_n w_n) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s - z_n)};$$
$$\tilde{k}_{sn}^{-+} = \left(c_s^- c_n^{+*}\right)/(\omega_n w_n) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s + z_n)}.$$

In this case, frequencies and amplitudes of coupled oscillations are also found from (3), (17).

4 Solution of the problem of scattering by an detuned DR system in a transmission line

Let us consider several special cases of scattering by systems of detuned DRs in various transmission lines that are of theoretical and practical interest.

Let us first consider two frequency-detuned bandstop filters made on different DRs (Fig. 2, a).

Here and below, circles of different sizes indicate detuned resonators; solid lines indicate the mutual coupling between the resonators k_{st}, and wavy lines indicate the coupling between the resonators along propagating waves $\tilde{k}_{st}$, as well as the possible coupling of the resonator with a regular transmission line $\tilde{k}_t$.

Let us assume that the natural frequency of the partial resonators of the first filter is $f_1 = 6,5$ GHz; and the second filter $f_2 = 7,5$ GHz. The quality factor of the dielectric of all resonators is the same: $Q^D = 2 \cdot 10^3$. The coupling coefficients of the resonators with the line $\tilde{k}_s = \tilde{k}_L$ and the coefficients of mutual coupling between adjacent filter resonators are also the same: $k_{st} = k_{12}$.

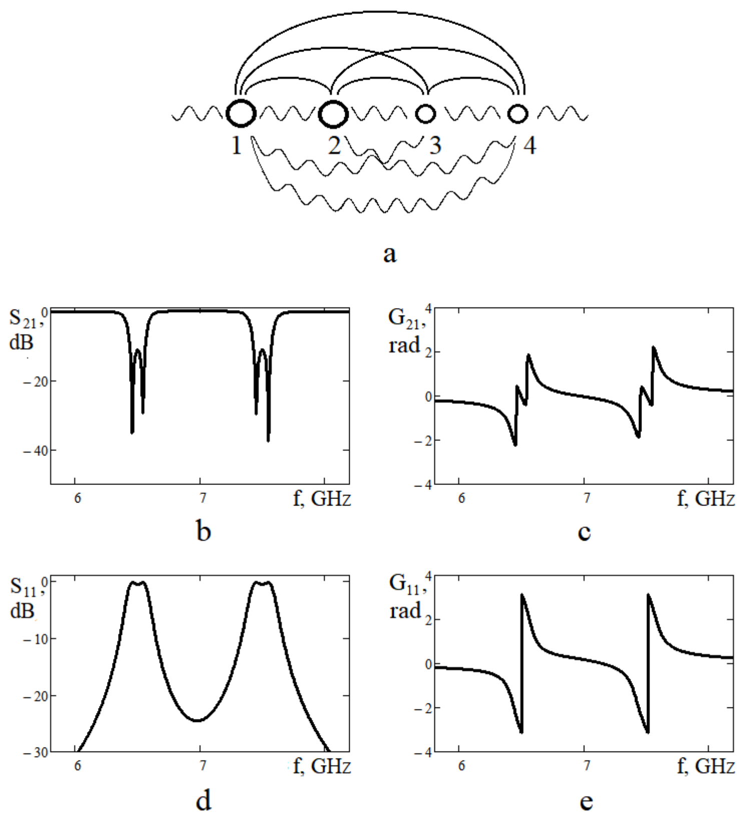

Fig. 2. 2+2 different DR system in a transmission line (a). Scattering characteristics of two different bandstop filters in the transmission line ($f_1 = 6,5$ GHz; $f_2 = 7,5$ GHz; $Q^D = 2 \cdot 10^3$; $\Gamma\Delta z_{12} = \pi/2$; distance between extreme filter resonators $\Gamma\Delta z_F = 2\pi$; coupling coefficients of resonators with the line $\tilde{k}_L = 0,015$; mutual coupling coefficients of adjacent resonators $k_{12} = 0,005$)

On Fig. 2, b–e shown the frequency characteristics of the scattering matrix on two bandstop filters made of two resonators, where $S_{21} = 20 \lg |T|$; $S_{11} = 20 \lg |R|$; $G_{21} = \arg(T)$; $G_{11} = \arg(R)$ (23).

An proposed scattering model allows us to simultaneously tune the characteristics of several bandstop and bandpass filters, controlling the transmission and reflection characteristics of the resonator system in the line.

On Fig. 3 shown an example of calculating the scattering characteristics of two bandpass filters placed parallel in a break of one line (a). Within the framework of the constructed scattering model, the coupling that occurs between the resonators of the input $1 - 1$ and output $N_1 - N_2$ transmission lines is taken into account.

The constructed model makes it possible to calculate and optimize the characteristics of scattering and more complex structures of detuned resonators. Consider, for example, a bandpass filter connected to regular transmission lines, at the input and output of which there are one or several resonators detuned in frequency (Fig. 4, a). Let's pretend that the distance between the outside filter resonators and the resonators in a regular line $\Gamma\Delta z_{12} \approx \Gamma\Delta z_{N-1,N} \approx \pi/2$.

The coupling parameters and natural frequencies of partial resonators 1, N, were selected in this way, to ensure the maximum steepness of the side slopes of the amplitude-frequency response of the bandpass filter (Fig. 4, b), while maintaining the linearity of the phase-frequency response (Fig. 4, c) in the passband.

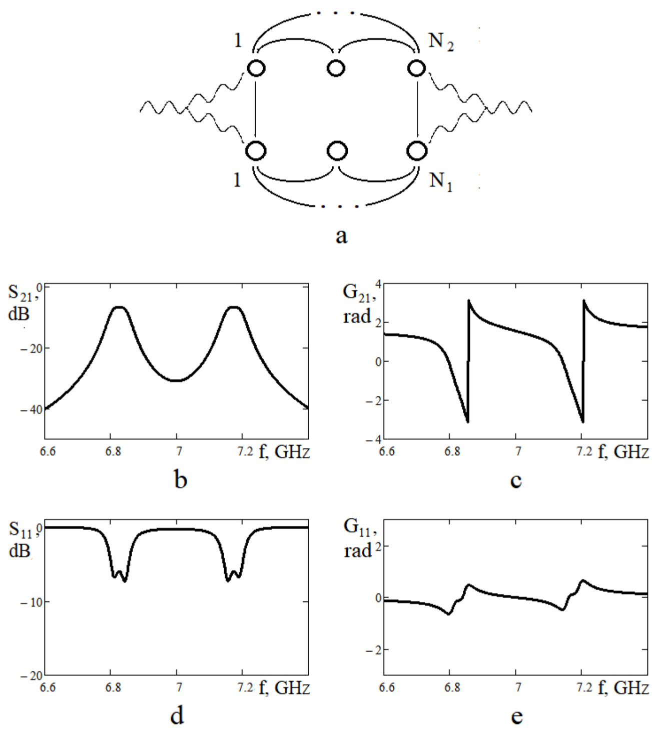

Fig. 3. 2+2 different DR system in a transmission line break (a). Scattering characteristics of two parallel bandpass filters ($f_1 = 6,85$ GHz; $f_2 = 7,15$ GHz; $Q^D = 2 \cdot 10^3$; coupling coefficients of resonators with the line: $\tilde{k}_L = 0,006$; mutual coupling coefficients of adjacent filter resonators: $k_{12} = 0,006$; mutual coupling coefficients of input (output) resonators: $k^F = -0,025$)

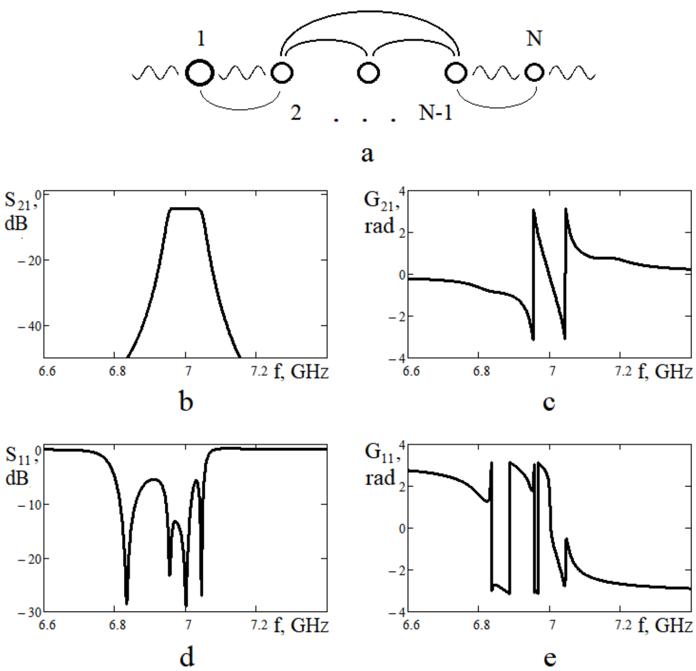

Fig. 4. 4+1+1 different DR system in a transmission line and transmission line break (a). Scattering characteristics of detuned in frequency bandpass and 2 bandstop filters ($f_1 = 6,83$ GHz; $f_2 = 7$ GHz; $f_3 = 7,17$ GHz; $Q^D = 2 \cdot 10^3$; coupling coefficients of resonators with the line: $\tilde{k}_L = 0,015$; mutual coupling coefficients of adjacent filter resonators: $k_{12} = 0,002$; mutual coupling coefficients of resonators in the line and outside filter resonators: $k^F = 0,0092$)

On Fig. 5 the scattering characteristics of a frequency-optimized elliptical bandpass filter are presented. In this case, the place where the N_1+1; N_1+2 resonators are "connected" to s, t resonators (a), as well as frequencies and mutual coupling coefficients were selected from the condition of ensuring maximum values of the steepness of the amplitude-frequency characteristic of the scattering matrix and minimizing attenuation (Fig. 5, b).

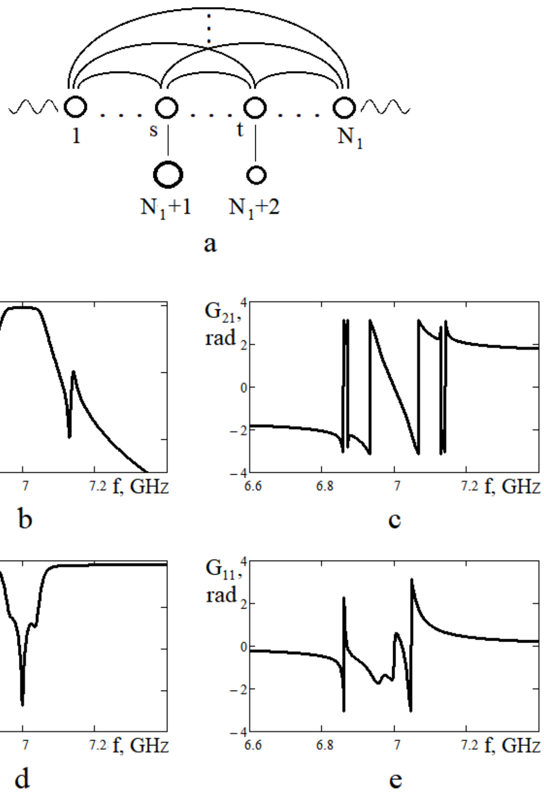

Fig. 5. Elliptical bandpass filter on 3+1+1 different DRs in a transmission line break (a). Scattering characteristics of the filter ($f_1 = 7$ GHz; $f_2 = 6,87$ GHz; $f_3 = 7,13$ GHz; $Q^D = 2\cdot 10^3$; coupling coefficients of resonators with the line: $\tilde{k}_L = 0,012$; mutual coupling coefficients of adjacent filter resonators: $k_{12} = 0,011$; mutual coupling coefficients of line resonators and shunt resonators: $k^F = 0,01$; $s = 1$; $t = 3$)

On Fig. 6 shows the example of the the scattering characteristics calculating of a demultiplexer built on two bandpass filters. Simultaneous optimization of filters also allows us to control the maximum attenuation between channels (Fig. 6, b), reflection coefficients (d), as well as phase-frequency characteristics (c-e).

The proposed theory of scattering by frequency-detuned DR systems can also be used to analyze the optimization of many other composite devices built on their basis.

Discussion and Conclusion

A general theory of scattering of electromagnetic waves by systems of detuned in frequency dielectric resonators has been developed. It is shown that the stray field can be expanded into a system of functions determined by the natural oscillations of the resonator system. Using perturbation theory, a system of equations that determines the frequencies and amplitudes of coupled oscillations of detuned resonators in a transmission line is obtained. A system of equations that determines the amplitudes of forced oscillations of resonators when line waves are incident on it has been found. In special cases of identical natural frequencies of partial resonators, the found equation systems coincide with obtained earlier.

Shown, that the developed theory makes it possible to simultaneously calculate and optimize the scattering characteristics of several bandpass and bandstop filters, multi-frequency filters, elliptical filters, multiplexers and other frequency-selective structures of complex structures containing detuned in frequency dielectric resonators.

The results of this study can be used in the design and optimization of various frequency-selective elements of communication devices in the microwave, terahertz, infrared and optical ranges.

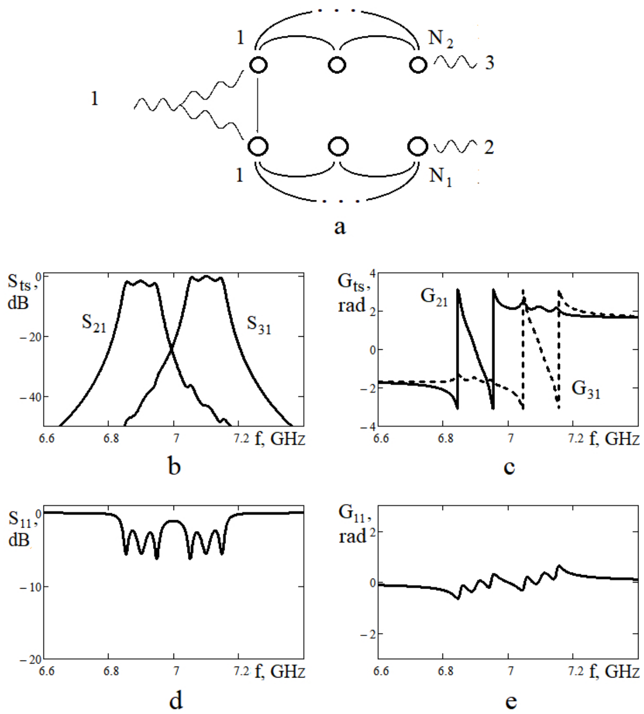

Fig. 6. Demultiplexer built on 3+3 different DR (a). Scattering characteristics of the detuned in frequency 2 bandpass filters ($f_1 = 6,9$ GHz; $f_2 = 7,1$ GHz; $Q^D = 2 \cdot 10^3$; coupling coefficients of resonators with the line: $\tilde{k}_L = 0,006$; mutual coupling coefficients of adjacent filter resonators: $k_{12} = 0,01$; mutual coupling coefficients of input 1-1 resonators (a): $k^{FF} = 0,005$)

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Розсіювання електромагнітних хвиль на розстроєних за частотами системах діелектричних резонаторів

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Розглядається загальна задача розсіювання на системі розстроєних за частотами зв'язаних діелектричних резонаторів (ДР), розташованих в одній або кількох лініях передачі. Поле, що описує власні коливання системи розстроєних ДР, розкладається по полю парціальних резонаторів. Виводиться система рівнянь, рішення якої дозволяє визначити частоти та амплітуди власних коливань системи. Показано, що отримана система рівнянь, шляхом алгебраїчних перетворень, може бути зведена до завдання на визначення власних функцій і власних значень скінченномірного оператора, що визначається через елементи оператора зв'язку розстроєних за частотами ДР. Зазначаються обмеження запропонованого методу розрахунку. Розв'язання задачі розсіювання розкладається по полю власних коливань системи розстроєних ДР. Отримано систему лінійних рівнянь для амплітуд вимушених коливань. Показано, що в окремому випадку однакових частот резонаторів знайдена система точно збігається з рівняннями, отриманими раніше для ДР різних видів. Знайдено загальні рішення для поля розсіювання на розстроєних за частотами резонаторах, що розташовуються у різних лініях передачі. Наводиться кілька прикладів розрахунку частотних залежностей матриці розсіювання для найцікавіших структур, що складаються зі зв'язаних розстроєних за частотами власних коливань, діелектричних резонаторів. Розраховуються частотні характеристики розсіювання двох режекторних фільтрів з різними смугами загородження, виконаних на розстроєних ДР в одній лінії передачі. Розраховані частотні залежності матриці розсіювання двох смугових фільтрів, розташованих паралельно між регулярними лініями. Розраховані характеристики розсіювання смугових фільтрів складних конструкцій, що містять різні ДР: смуговий фільтр, побудований на системі зв'язаних ДР у розриві лінії передачі та кількох ДР у регулярній лінії, а також еліптичний смуговий фільтр. Можливості запропонованого методу демонструються на прикладі оптимізації характеристик розсіювання демультиплексера, побудованого на основі двох смугових фільтрів із різними частотними смугами пропускання. Запропонований метод дозволяє також розраховувати характеристики розсіювання смугових та режекторних фільтрів з кількома смугами робочих частот, що застосовуються у сучасних системах зв'язку.

Ключові слова: розсіювання; розстроєний діелектричний резонатор; матриця розсіювання; смуговий фільтр; режекторний фільтр; еліптичний фільтр; демультиплексер